Central University of Punjab, Bathinda

Course Scheme & Syllabus

for

Ph. D. Course Work

in

Mathematics / Statistics

Session 2019-21

Department of Mathematics and Statistics

Ph.D. (Mathematics/Statistics) Programme

Vision:

Department of Mathematics and Statistics aims at holistic development through academic excellence, problem solving skills, analytical thinking and employment opportunities.

Mission:

The mission of the department is to establish as a well known centre of teaching and learning recognized internationally. The aim of the department is to provide an academic environment where students can compete at national and international level in mathematics and statistics through research and teaching.

1. **Course Offered:**
   Ph.D. Mathematics
   Ph. D. Statistics

2. **Eligibility Criteria:**

Ph. D.	Mathematics	1. Master degree in Mathematics with minimum 60% marks (55% for SC/ST/PWD/OBC (non-creamy layer)) from a recognized Indian or Foreign University 2. UGC-CSIR NET-JRF qualified candidates in Mathematics Science or having any National/State level fellowships like RGNF/MANF/NBHM etc.
Ph. D.	Statistics	1. Master degree in Statistics with minimum 60% marks (55% for SC/ST/PWD/OBC (non-creamy layer)) from a recognized Indian or Foreign University 2. UGC-CSIR NET-JRF qualified candidates in Mathematics Science or having any National/State level fellowships like RGNF/MANF/NBHM etc.

3. **Admission Process:**
   Admission to this course is based on Central Universities Common Entrance Test (CUCET) and consequent interview at Department level.

4. **Duration of Course**
   Minimum 3 years (6 Semesters)

5. **Number of Seats to be Offered (Intake)**
   As per availability of seats.

6. **Reservation:**
   As per Government of India rules applicable from time to time

7. **Course Structure and Detailed Syllabus**

   As per details given below.

Course structure for Ph.D. Course work in Mathematics / Statistics

Students can move into the Ph.D. programme after successful completion of Ph. D. Course work during first two semesters, provided they meet the requirements specified by the university.

Structure for course work for PhD in Mathematics / Statistics

	Semester I						
	S. No.	**Subject Code**	**Subject Name**	**Credit Hours**			**Maximum Marks**
				Lecture	Practical	Credits	
Compulsory Courses	1.	PHDMS.701	Research Methodology	3		3	75
	2	PHDMS.702	Computer Applications	2	1	3	75
	2.	PHDMS.703	Review Writing and Seminar		4	2	50
	Opt any two out of the following elective courses offered						
	3.	PHDMS.704	Real Analysis	4		4	100
	4.	PHDMS.705	Linear Algebra	4		4	100
	5.	PHDMS.706	Symmetries and Differential Equations	4		4	100
	6.	PHDMS.707	Fractional Calculus	4		4	100
	7.	PHDMS.708	Advanced Partial Differential Equations	4		4	100
	8.	PHDMS.709	Advanced Numerical Analysis	4		4	100
	9.	PHDMS.710	Operations Research	4		4	100
	10.	PHDMS.711	Number Theory	4		4	100
	11.	PHDMS.712	Advanced Algebra	4		4	100
	12.	PHDMS.713	Functional Analysis	4		4	100

	13.	PHDMS.714	Advanced Complex Analysis	4		4	100
	14.	PHDMS.715	Differential Equations and Boundary-Value Problems	4		4	100
	15.	PHDMS.716	Potential Flow of Fluids and Water-wave Theory	4		4	100
	16.	PHDMS.717	Topology	4		4	100
	17.	PHDMS.718	Differential Geometry	4		4	100
	18.	PHDMS.719	Differential Topology	4		4	100
	19.	PHDMS.720	Algebraic Topology	4		4	100
	20.	PHDMS.721	Riemannian Geometry	4		4	100
	21.	PHDMS.722	Riemann - Finsler Geometry	4		4	100
	22.	PHDMS.723	General Relativity	4		4	100
	23.	PHDMS.724	Probability Theory	4		4	100
	24.	PHDMS.725	Stochastic Processes and Queuing Theory	4		4	100
	25.	PHDMS.726	Reliability Theory	4		4	100
	26.	PHDMS.727	Sampling Theory	4		4	100
	Total					**16**	**400**

Syllabi for Ph. D. Course work
Semester I

Course Title: Research Methodology
Course Code: PHDMS.701
Total Hours: 45

L	T	P	Cr
3	0	0	3

Learning Outcomes: The objective of this course is to ensure that a student learns basis of scientific research. Also the student will develop understanding of computer hardware systems and its few basic applications.

Unit-I **12 Hours**

Introduction: Meaning, Objectives, Characteristics, Significance, and Types of Research.

Formulating Research Problem: Understanding a Research Problem, Selecting the Research Problem, Steps in Formulation of a Research Problem, Formulation of Research Objectives, and Construction of Hypothesis.

Unit-II **11 Hours**

Literature Survey and Review: Meaning of Literature Survey and Review, Sources of Literature, Methods of Literature Review, and Techniques of Writing the Reviewed Literature.

Plagiarism: Plagiarism, definition, Search engines, regulations, policies and documents/thesis/manuscripts checking through software, Knowing and Avoiding Plagiarism during documents/thesis/manuscripts/scientific writing.

Unit-III **11 Hours**

Installation of the software LaTeX, Understanding LaTeX compilation and LaTeX editors, Basic syntax, Writing mathematical equations, Matrices, Tables, Inclusion of graphics into LaTeX file.

Page configurations: Title, Abstract, Keywords, Chapter, Sections and Subsections, References and their citations, Labeling of equations, Table of contents, List of figures, List of tables, Page numbering, Generating index.

Packages: amsmath, amssymb, amsthm, amsfonts, hyperrefer, graphic, color, xypic, latexsym, natbib, setspace, multicol, subcaption, url, verbatim, tikz, and geometry.

Classes: Article, Report, Book, Letter, Slides, Beamer.

Unit-IV **11 Hours**

Report Writing: Types of Reports – Technical and Popular Reports, Significance of Report Writing, Different Steps in Writing Report, Art of Writing Research Proposals, Research Papers, Project Reports, and Dissertations/Thesis; Basics of Citation and Bibliography/Reference Preparation Styles; Report Presentation: Oral and Poster Presentations of Research Reports.

Suggested Readings:

1. Kothari, C.R. and G. Garg (2014): *Research Methodology*: Methods and Techniques, 3rd ed., New Age International Pvt. Ltd. Publisher
2. Kumar, R. (2014): *Research Methodology – A Step-By-Step Guide for Beginners*, 4th ed., Sage Publications

3. J. Anderson, *Thesis and Assignment Writing*, 4th ed., Wiley, USA, 2001.
4. Catherine Dawson, *Practical Research Methods,* New Delhi, UBS Publishers' Distributors, 2014.
5. L. Lamport. LATEX: A Document Preparation System, User's Guide and Reference Manual. 2nd Edition, Addison Wesley, New York, 1994.
6. Copyright Protection in India [website: http:copyright.gov.in].
7. World Trade Organization [website: www.wto.org].
8. Wadedhra B.L. Law Relating to Patents, Trademarks, Copyright Design and Geographical Indications. Universal Law Publishing, New Delhi. Latest Edition.

Course Title: Computer Applications
Course Code: PHDMS.702
Total Hours: 30

L	T	P	Cr
2	0	1	3

Learning outcomes:
Upon successful completion of this course, the student will be able to:

1. Use different operating system and their tools easily.
2. Use word processing software, presentation software, spreadsheet software and latex.
3. Understand networking and internet concepts.
4. Use computers in every field like teaching, industry and research.

Unit-I **7 Hours**

Word Processing using MS Word: Text creation and Manipulation; Table handling; Spell check, Hyper-linking, Creating Table of Contents and table of figures, Creating and tracking comments, language setting and thesaurus, Header and Footer, Mail Merge, Different views, Creating equations, Page setting, Printing, Shortcut keys.

Unit-II **8 Hours**

Presentation Tool: Creating Presentations, Presentation views, working on Slide Transition, Making Notes Pages and Handouts, Drawing and Working with Objects, Using Animations, Running and Controlling a Slide Show, Printing Presentations, Shortcut keys.

Unit-III **7 Hours**

Spread Sheet: Entering and editing data in cell, Basic formulas and functions, deleting or inserting cells, deleting or inserting rows and columns, printing of Spread Sheet, Shortcut keys.

Unit-IV **8 Hours**

Use of Computers in Education and Research: Data analysis tools, e-Library, Search engines related to research, Research paper editing tools like Latex.

Suggested Readings:

1. Sinha, P.K. Computer Fundamentals. BPB Publications.
2. Goel, A., Ray, S. K. 2012. Computers: Basics and Applications. Pearson Education India.

3. Microsoft Office Professional 2013 Step by Step https://ptgmedia.pearsoncmg.com/images/9780735669413/samplepages /97807356694 13.pdf

Course Title: Review Writing and Presentation
Course Code: PHDMS.703
Total Hours: 60

L	T	P	Cr
0	0	4	2

Learning Outcomes: The objective of this course would be to ensure that the student learns the aspects of the Review writing and seminar presentation. Herein the student shall have to write a review of existing scientific literature with simultaneous identification of knowledge gaps that can be addressed through future work.

The evaluation criteria for "Review Writing and Presentation" shall be as follows:

S. No.	Criteria	Marks
1.	Literature review report	20
2.	Content of presentation	10
3.	Presentation Skills	10
4.	Handling of queries	10
Total		**50**

Course Title: Real Analysis
Course Code: PHDMS.704
Total Lectures: 60

L	T	P	Cr
4	0	0	4

Learning outcomes: The aim of this course is to make the students learn fundamental concepts of metric spaces, Riemann-Stieltjes integral as a generalization of Riemann Integral, Sequence and series of functions and some basic theorems.

Unit-I **15 Hours**

Set Theory: Finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum

Metric spaces: Definition and examples, Open and closed sets, Compact sets, Elementary properties of compact sets, k- cells, Compactness of k-cells, Compact subsets of Euclidean space $\Re^k$, Bolzano Weierstrass theorem, Heine Borel theorem, Perfect sets, Cantor set, Separated sets, Connected sets in a metric space, Connected subsets of real line.

Unit-II **15 Hours**

Sequences in Metric spaces: Convergent sequences, Subsequences, Cauchy sequences, Complete metric space, Cantor's intersection theorem, Category of a set and Baire's category theorem. Examples of complete metric space, Banach contraction principle.

Unit-III **15 Hours**

Continuity: Limits of functions (in Metric spaces), Continuous functions, Continuity and compactness, Continuity and connectedness, Discontinuities, Monotonic functions, Uniform continuity.

Riemann Stieltje's Integral: Definition and existence of Riemann Stieltje's integral, Properties of integral. Integration and Differentiation. Fundamental Theorem of Calculus, 1st and 2nd Mean Value Theorems of Riemann Stieltje's integral.

Unit-IV **15 Hours**

Sequences and series of functions: Problem of interchange of limit processes for sequences of functions, Uniform convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, equicontinuous families of functions, Stone Weierstrass Theorem.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:

1. Tom M. Apostol, *Mathematical Analysis*, Addition –Wesley, USA, 1981.
2. R. G. Bartle, *The Elements of Real Analysis*, John Willey and Sons, New York, 1976.
3. A. Kumar and S. Kumaresan, *A Basic Course in Real Analysis*, Narosa, Publishing House, New Delhi, 2014.
4. W. Rudin, *Principles of Mathematical Analysis,* 3rd Edition, McGraw Hill, Kogakusha, International student Edition, 1976.
5. E. C. Titchmarsh, *The Theory of functions,* Oxford University Press, Oxford, 2002.

Course Title: Linear Algebra
Course Code: PHDMS.705
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes: The concepts and techniques from linear algebra are of fundamental importance in many scientific disciplines. The main objective is to introduce basic notions in linear algebra that are often used in mathematics and other sciences. The emphasis will be to combine the abstract concepts with examples in order to intensify the understanding of the subject.

Unit I **15 Hours**

Vector spaces, Subspaces, Linear dependence and independence, Basis and dimensions, Coordinates, Linear transformations, Algebra of linear transformations, Isomorphism, Matrix representation of a linear transformation, Change of basis, Rank and nullity of a linear transformation. Linear functionals, Dual spaces, Transpose of a linear transformation.

Unit II **16 Hours**

Characteristic polynomial and minimal polynomial of a linear transformation, Characteristic values and Characteristic vectors of a linear transformation, Cayley Hamilton theorem, Invariant subspaces, Diagonalization and triangulation of a matrix, Direct sum of subspaces, Invariant Direct sums, Characteristic polynomial and minimal polynomial of block matrices.

Unit III **15 Hours**

Cyclic subspaces and Annihilators, Canonical forms: Jordan canonical forms, rational canonical forms. Quotient spaces, Bilinear forms, Symmetric and skew-Symmetric bilinear forms, Sylvester's theorem, quadratic forms, Hermitian forms. Reduction and classification of quadratic forms.

Unit IV **14 Hours**

Inner product spaces. Norms and distances, Orthonormal basis, Orthogonality, Schwartz inequality, The Gram-Schmidt orthogonalization process. Orthogonal and positive definite matrices. The Adjoint of a linear operator on an inner product space, Normal and self-adjoint operators, Unitary and orthogonal operators.

Suggested Readings:

1. P. B. Bhattacharya, S.K. Jain and S.R. Nagpaul, *Basic Abstract Algebra,* Wiley Eastern, Delhi, 2003.
2. V. Bist and V. Sahai, *Linear Algebra,* Narosa, Delhi, 2002.
3. J. Gilbert and L. Gilbert, *Linear Algebra and Matrix Theory,* Cengage Learning, 2004.
4. K. Hoffman and R. Kunze: *Linear Algebra,* 2nd Edition, Pearson Education (Asia) Pvt. Ltd/ Prentice Hall of India, 2004.
5. I. S. Luthar and I. B. S. Passi, *Algebra: Rings*, Volume 2, Narosa Publishing House, 2000.

Course Title: Symmetries and Differential Equations
Course Code: PHDMS.706
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes: The objective of this course is to ensure that a student learns basis of Dimensional analysis and Lie group of transformations. Also the student will develop understanding of its few basic applications for solving ordinary and partial differential equations.

Unit I **15 Hours**

Dimensional Analysis: Buckingham Pi-theorem, Assumptions behind dimensional analysis, Conclusions from dimensional analysis, Proof of the Buckingham Pi-theorem and examples, Application of dimensional analysis to partial differential equations, Generalization of dimensional analysis, Invariance of partial differential equations under scaling of variables

Unit II **15 Hours**

Lie Group of Transformations: Groups, Examples of groups, Groups of transformations, One-parameter Lie group of transformations, Examples of one-parameter Lie groups of transformations, Infinitesimal transformations: First fundamental theorem of Lie, Infinitesimal generators, Invariant functions

Unit III **15 Hours**

Canonical coordinates, Invariant surfaces, Invariant curves, Invariant points, Extended transformations: Extended group transformations-one dependent and one independent variable, Extended infinitesimal transformations-one dependent and one independent variable, Extended transformations-one dependent and n independent variables

Unit IV **15 Hours**

Multi-parameter Lie groups of transformations; Lie algebras, r-parameter Lie groups of transformations, Lie algebras, Examples of Lie algebras, Solvable Lie algebras

Suggested Readings:

1. G. W. Bluman and A. C. Anco, *Symmetry and Integration Methods for Differential Equations*, Appl. Math. Sci., 154, Springer, New York, 2002.
2. G. W. Bluman and S. Kumei, *Symmetries and Differential Equations*, Appl. Math.Sci., Springer-Berlin, 1989.
3. P. J. Olver, *Applications of Lie Groups to Differential Equations*, Springer-Verlag, New York 1993.
4. L. V. Ovsiannikov, *Group Properties of Differential Equations*, Novosibirsk, Moscow, 1962.

Course Title: Fractional Calculus
Course Code: PHDMS.707
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes: The objective of this course is to ensure that a student learns basis of fractional calculus. Also the student be able study the fractional differential equations which will use in research field of fractional calculus.

Unit I **16 Hours**

Special Functions of Fractional Calculus: Gamma function, Some properties of Gamma function, Beta function, Contour integral representation. Fractional derivatives and integrals, GrunwaldLetnikov Fractional derivatives, Riemann-Liouville fractional derivatives, Caputo's fractional derivative, The Leibniz rule for fractional derivatives, Geometric and physical interpretation of fractional integration and fractional differentiation.

Unit II **14 Hours**

Sequential fractional derivatives. Left and right fractional derivatives. Properties of fractional derivatives. Laplace transforms of fractional derivatives. Fourier transforms of fractional derivatives. Mellin t
transforms of fractional derivatives.

Unit III **15 Hours**

Linear Fractional Differential Equations: Fractional differential equation of a general form. Existence and uniqueness theorem as a method of solution. Dependence of a solution on initial conditions. The Laplace transform method. Standard fractional differential equations. Sequential fractional differential equations.

Unit IV **15 Hours**

Fractional Differential Equations: Introduction, Linearly independent solutions, Solutions of the homogeneous equations, Solution of the non-homogeneous fractional differential equations, Reduction of fractional differential equations to ordinary differential equations. Semi differential equations

Suggested Readings:

1. K. B. Oldham & J. Spanier, *The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order*, Dover Publications Inc, 2006.
2. K. S. Miller & B. Ross., *An Introduction to the Fractional Calculus and Fractional Differential Equations Hardcover*, Wiley Blackwell, 1993.
3. I. Podlubny, *Fractional Differential Equations*, Academic Press, 1998

Course Title: Advanced Partial Differential Equations
Course Code: PHDMS.708
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes: The objective of this course is to equip the students with knowledge of some advanced concepts related to partial differential equations and to understand some basic approaches to mathematical oriented PDEs.

Unit-I **16 Hours**

Distribution: Test functions and distributions, examples, operations on distributions, supports and singular supports, convolution, fundamental solutions, fourier transform, Schwartz space, tempered Distributions.

Sobolev Spaces: Basic properties, approximation by smooth functions, extension theorems, compactness theorems, dual spaces, functional order spaces, trace spaces, trace theory, inclusion theorem.

Unit-II **15 Hours**

Weak Solutions of Elliptic Boundary Value Problems: variational problems, weak formulation of elliptic PDE, regularity, Galerkin method, Maximum principles, eigenvalue problems, introduction to finite element methods.

Unit-III **14 Hours**

Evolution Equations: unbounded linear operators, C_0 – semigroups, Hille-Yosida theorem, contraction semigroup on Hilbert spaces, heat equation, wave equation, Schrodinger equation, inhomogeneous equations.

Unit-IV **15 Hours**

Calculus of Variations: Euler-Lagrange equation, second variation, existence of minimizers (coactivity, cower semi-continuity, convexity), regularity, constraints (nonlinear eigenvalue problems, variational inequalities, harmonic maps, incompressibility), critical points (mountain pass theorem and applications to elliptic PDE).

Suggested Books:

1. S. Kesavan, *Topics in Functional Analysis and Application*, Wiley-Eastern, New International, New Delhi, 1999.
2. L. C. Evans, *Partial Differential Equations. Graduate Studies in Mathematics*, American Mathematical Society, 2nd Edition, Indian Reprint, 2014.
3. Rao, K. S., *Introduction to Partial Differential Equation*, 2nd Edition, PHI Learning Pvt. Ltd. 2010.
4. Amarnath, T., *An Elementary Course in Partial Differential Equations*, 2nd Edition, Narosa Publishing House 2012.
5. Sneddon, I. N., *Elements of Partial Differential Equations*, McGraw-Hill Book Company, New York 1988.

Course Title: Advanced Complex Analysis
Course Code: PHDMS.709

L	T	P	Cr
4	0	0	4

Total Hours: 60

Learning Outcomes:

This course is designed to enable the readers to understand further deeper topics of Complex Analysis and will provide basic topics needed for students to pursue research in pure Mathematics.

Unit–I **16 Hours**

Harmonic function: Definition, relation between a harmonic function and an analytic function, examples, harmonic conjugate of a harmonic function, poisson's integral formula, mean value property, the maximum & minimum principles for harmonic functions, Dirichlet problem for a disc and uniqueness of its solution, characterization of harmonic functions by mean value property.

Unit–II **16 Hours**

Analytic continuation: Direct analytic continuation, analytic continuations along arcs, Homotopic curves, the monodromy theorem, analytic continuation via reflection. Harneck's principle. Open mapping theorem, normal families, The Riemann mapping theorem, Picard's theorem.

Unit–III **14 Hours**

Weierstrass Elliptic functions: Periodic functions, simply periodic functions, fundamental period, Jacobi's first and second question, doubly periodic functions, elliptic functions, pair of Primitive periods, congruent points, first and second Liouville's theorem, relation between zeros and poles of an elliptic function, definition of Weierstrass elliptic function (z) and their properties, the differential equation satisfied by (z) [i.e., the relation between (z) and $(\bar{z})$], Integral formula for (z), addition theorem and duplication formula for (z).

Unit- IV **13 Hours**

Weierstrass Zeta function: Weierstrass zeta function and their properties, quasi periodicity of (z), Weierstrass sigma function (z) and their properties, associated sigma functions.

Suggested Readings:

1. J. B. Conway, *Functions of One Complex Variable*, 2nd Edition, Springer-Verlag International, USA, 1978.
2. L.V. Ahlfors, *Complex Analysis: An Introduction to the Theory of Analytic Functions of One Complex Variable*, 3rd Edition, McGraw-Hill, Higher Education, New Delhi, 1979.
3. S. Lang, *Complex Analysis*, 4th Edition, Springer, New York, 2003.
4. R. Walter, *Real and Complex Analysis*, 3rd Edition, McGraw-Hill Book Co., New Delhi, 1986.

5. S. Ponnusamy, *Foundations of Complex Analysis*, 2nd Edition, Narosa Publication House, New Delhi, 1995.

Course Title: Operations Research
Course Code: PHDMS.710
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes:
The objective of this course is to acquaint the students with the concept of convex sets, their properties, Linear and nonlinear programming problems. The results, methods and techniques contained in this paper are very well suited to the realistic problems in almost every area

Unit-I **15 Hours**
Operations Research and its Scope, Necessity of Operations Research in industry Mathematical formulation of linear programming problem, Linear Programming and examples, Convex Sets, Hyper plane, Open and Closed half-spaces, Feasible, Basic Feasible and Optimal Solutions, Extreme Point & graphical methods. Simplex method, Big-M method, Two phase method, Determination of Optimal solutions, Unrestricted variables.

Unit-II **15 Hours**
Duality theory, Dual linear Programming Problems, Fundamental properties of dual problems, Complementary slackness, Unbounded solution in Primal. Dual Simplex Algorithm, Sensitivity analysis.

Unit-III **15 Hours**
The General transportation problem, Duality in transportation problem, Loops in transportation tables, Solution of transportation problem, Test for optimality, Degeneracy, Transportation algorithm (MODI method), Minimization transportation problem. Assignment Problems: Mathematical formulation of assignment problem, Hungarian method for solving assignment problem, Traveling salesman problem and Sequencing.

Unit -IV **15 Hours**
Replacement problem, replacement of items that Deteriorate, replacement of items that fails completely. Job Sequencing Problems; Introduction and assumption, Processing of n jobs through two machines, Processing of n jobs through three machines and m machines, Processing two jobs through n machines.

Suggested Readings:
1. N. S. Kambo, *Mathematical Programming Techniques*, Affiliated East- West Press Pvt. Ltd., 1984, Revised Edition, New Delhi, 2005.

2. G. Hadley, *Linear Programming*, Narosa Publishing House, New Delhi, 1987.
3. S. M. Sinha, *Mathematical Programming, Theory and Methods,* Delhi: Elsevier, 2006.
4. K. Swarup, P. K. Gupta, and M. Mohan, *Operations Research*, Sultan Chand & Sons, New Delhi, 2001.
5. H. A. Taha, *Operations Research - An Introduction*, Macmillan Publishing Company Inc., New York, 2006.

Course Title: Number Theory
Course Code: PHDMS.711
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes:
The objective of this course is to teach the fundamentals of different branches of Number Theory, namely, Geometry of Numbers and Analytic Number Theory.

Unit I **15 Hours**
Divisibility of Integers, Greatest common divisor, Euclidean algorithm. The fundamental theorem of arithmetic, Congruences, Residue classes and reduced residue classes.

Unit II **15 Hours**
Indices and its applications, Quadratic residues, Euler's criterion, Product of quadratic residues and quadratic non-residues, The Legendre symbol and its properties, Gauss's lemma, Quadratic reciprocity law, Jacobi symbol and its properties.

Unit III **15 Hours**
Chinese remainder theorem, Fermat's little theorem, Wilson's theorem, Euler's theorem. Arithmetic functions $\sigma(n)$, $d(n)$, $\tau(n)$, $\mu(n)$, Order of an integer modulo n, primitive roots for primes, composite numbers having primitive roots.

Unit IV **15 Hours**
Representation of an integer as a sum of two and four squares. Diophantine equations ax +by =c, x2+y2=z2 and its application to x4+y4=z4. Farey sequences, Continued fractions.

Suggested Readings:
1. David, M. Burton, *Elementary Number Theory*, Tata McGraw-Hill,7th Edition, New Delhi, 2012.
2. I. Niven, S. Zuckerman, and H. L. Montgomery, *Introduction to Number Theory*, Wiley Eastern, 1991.
3. T. N. Apostol, *Introduction to Analytic Number Theory*, Springer Verlag, 1976.
4. G. H. Hardy and E. M. Wright, *An Introduction to the Theory of Number*, Oxford

Univ. Press, U.K., 2008.
5. W. W. Adams and L. J. Goldstein, *Introduction to Number Theory*, Prentice Hall Inc., 1976.

Course Title: Advanced Algebra
Course Code: PHDMS.712
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes:
This course is for students who wish to pursue research work in Algebra.

Unit I **14 Hours**
Groups: Groups: Jordan Holder theorem; solvable groups; symmetric and alternating groups; nilpotent groups; groups acting on sets; Sylow theorems; free groups.

Unit-II 16 Hours
Rings and Modules: Noetherian and Artinian rings and modules; semi-simple rings; Hilbert basis theorem; Principal ideal domains and unique factorisation domains; modules over PID; linear algebra and Jordan canonical form; structure theorems for semi-simple rings. Representation theory of finite groups.

Unit-III 14 Hours
Field Theory: Basic concepts of field theory, Steinitz theorem, Extension of fields, algebraic and transcendental extensions. Algebraically closed fields, Splitting fields, Separable and inseparable extensions, Normal extension, Multiple roots, Finite fields, Perfect fields.

Unit-IV 16 Hours
Galios Theory: Automorphism groups, Fixed fields, Galois extensions, The fundamental theorem of Galois theory, Cyclotomic extensions, and Cyclic extensions, Applications of cyclotomic extensions and Galois theory to the constructability of regular polygons, Solvability of polynomials by radicals.

Suggested Readings:
1. M. Artin, *Algebra*, 2nd Edition, Prentice Hall of India, 2011.
2. P. B. Bhattacharya, S.K. Jain and S.R. Nagpaul, *First Course in Linear Algebra*, Wiley Eastern, Delhi, 2008.
3. D. S. Dummit and R. M. Foote, *Abstract Algebra,* 3rd Edition, John Wiley, 2011.
4. J. P. Escofier, *Galois Theory*, Springer-Verlag, 2000.
5. J. Gilbert and L. Gilbert, *Linear Algebra and Matrix Theory*, Academic Press, 2004.
6. B. Hartley and T. O. Hawkes, *Rings, Modules and Linear Algebra*, Chapman and Hall, 1970.

7. I. N. Herstein, *Topics in Algebra*,2nd Edition, Wiley Eastern Limited, New Delhi, 2006
8. I. S. Luthar and I. B. S. Passi, *Algebra Vol III: Modules*, Narosa Publishing House, 2002.
9. C. Musili, *Rings and Modules*,2nd Revised Edition, Narosa Publishing House, New Delhi, 1994.
10. I. B. S. Passi and I. S. Luthar, *Algebra: Volume 4: Field Theory*, Narosa Publishing House, New Delhi, 2010.

Course Title: Functional Analysis
Course Code: PHDMS.713
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning outcomes: The objective of this course is to introduce basic concepts, methods of Functional Analysis and its Applications. It is a first level course in Functional Analysis.

Unit-I **14 Hours**

Fundamentals of Normed Linear Spaces: Nomed Spaces, with examples of Function spaces L^P([a,b]) ,C([a,b]) and C^1([a,b]), Sequence Spaces l^p, c , c_0, c_{00} Banach spaces and examples, finite dimensional normed spaces and subspaces, compactness and finite dimension

Unit-II **15 Hours**

Bounded linear transformations, Normed linear spaces of bounded linear transformations, Dual spaces with examples.

Three Main Theorems on Banach Space: Banach Steinhauns theorem (Uniform boundedness theorem) and some of its consequences, Open mapping and closed graph theorems.

Unit-III **14 Hours**

Hahn-Banach theorem for real linear spaces and its consequences, Reflexive spaces, Solvability of linear equations in Banach spaces.

Unit-IV **17 Hours**

Geometry of Hilbert spaces: Inner product spaces, orthonormal sets, Approximation and optimization, Projections and Riesz Representation theorem. Bounded Operators on Hilbert spaces: Bounded operators and adjoints; normal, unitary and self adjoint operators, Spectrum and Numerical Range.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:

1. S. K. Berberian, *Introduction to Hilbert Spaces,* AMS Chelsea Publishing, Rhode Island, 1996.
2. C. Goffman, and G. Pedrick, *First Course in Functional Analysis*, Prentice Hall of India, New Delhi, 1983.
3. E. Kreyszig, *Introductory Functional Analysis with Application*, Willey, 2007.
4. B. V. Limaye, *Functional Analysis,* New Age International (P) Ltd, New Delhi, 1996.
5. F. K. Riesz, and B. S. Nagy, *Functional Analysis*, Dover Publications, 1990.
6. A. H. Siddiqui, *Functional Analysis*, Tata-McGraw Hill, New Delhi, 1987.
7. W. Rudin, *Functional Analysis*, McGraw Hill Education; 2 edition, 2017.

Course Title: Advanced Complex Analysis
Course Code: PHDMS.714
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes: This course is designed to enable the readers to understand further deeper topics of Complex Analysis and will provide basic topics needed for students to pursue research in pure Mathematics.

Unit–I **16 Hours**

Harmonic function: Definition, relation between a harmonic function and an analytic function, examples, harmonic conjugate of a harmonic function, poisson's integral formula, mean value property, the maximum & minimum principles for harmonic functions, Dirichlet problem for a disc and uniqueness of its solution, characterization of harmonic functions by mean value property.

Unit–II **16 Hours**

Analytic continuation: Direct analytic continuation, analytic continuations along arcs, Homotopic curves, the monodromy theorem, analytic continuation via reflection. Harneck's principle. Open mapping theorem, normal families, The Riemann mapping theorem, Picard's theorem.

Unit–III **14 Hours**

Weierstrass Elliptic functions: Periodic functions, simply periodic functions, fundamental period, Jacobi's first and second question, doubly periodic functions, elliptic functions, pair of Primitive periods, congruent points, first and second Liouville's theorem, relation between zeros and poles of an elliptic function, definition of Weierstrass elliptic function (z) and their properties, the differential equation satisfied by (z) [i.e., the relation between (z) and ($\bar{z}$)], Integral formula for (z), addition theorem and duplication formula for (z).

Unit- IV **13 Hours**

Weierstrass Zeta function: Weierstrass zeta function and their properties, quasi periodicity of (z), Weierstrass sigma function (z) and their properties, associated sigma functions.

Suggested Readings:

1. J. B. Conway, *Functions of One Complex Variable*, 2nd Edition, Springer-Verlag International, USA, 1978.
2. L.V. Ahlfors, *Complex Analysis: An Introduction to the Theory of Analytic Functions of One Complex Variable*, 3rd Edition, McGraw-Hill, Higher Education, New Delhi, 1979.
3. S. Lang**,** *Complex Analysis*, 4th Edition, Springer, New York, 2003.
4. R. Walter, *Real and Complex Analysis*, 3rd Edition, McGraw-Hill Book Co., New Delhi, 1986.
5. S. Ponnusamy, *Foundations of Complex Analysis*, 2nd Edition, Narosa Publication House, New Delhi, 1995.

Course Title: Differential Equations and Boundary-Val Problems
Course Code: PHDMS.715

L	T	P	Cr
4	0	0	4

Total Hours: 60

Learning Outcomes: The objective of this course is to ensure that a student learns basics differential equations and boundary value problems.

Unit I **16 Hours**

Existence and uniqueness of solutions of ODEs, power series solution, singular points, some special functions. Non-linear system of ODE: preliminary concepts and definitions, the fundamental existence-uniqueness results, dependence on initial conditions and parameters, the maximum interval of existence.

Unit II **15 Hours**

Linearlization, stability and Liapunov functions, saddle, nodes, foci and centers, normal form theory and Hamiltonian systems. Boundary value problems: Green's function method, Sturm-Liouville problem.

Unit III **14 Hours**

First-order PDEs, cauchy problem, method of characteristics, second-order PDEs, classification, characteristics and canonical forms. Elliptic boundary value problems: maximum principle, Green's function,

Unit IV **15 Hours**

Sobolev spaces, variational formulations, weak solutions, Lax-Milgram theorem, trace theorem, Poincarénequality, energy estimates, Fredholm alternative, regularity estimates, system of conservation laws, entropy criteria.

Suggested Readings:
1. L. Perko, *Differential Equations and Dynamical Systems*, 3rd Edition Springer-Verlag New York, 2001.
2. J. Guckenheimer, P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, 1st Edition, Springer-Verlag, New York, 1983.
3. S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, 2nd Edition, Springer-Verlag, New York, 1990.
4. L. C. Evans, *Partial Differential Equations, Graduate Studies in Mathematics*, Vol. 19, American Mathematical Society, Providence, 1998.
5. R. C. McOwen, *Partial Differential Equations-Methods and Applications*, 2nd Edition Pearson Education Inc., Indian Reprint 2002.
6. S. J. Farlow, *Partial Differential Equations for Scientists and Engineers*, Dover Publications, New York, 1982.

Course Title: Potential Flow of Fluids and Water-Wave Theory
Course Code: PHDMS.716
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes: The objective of this course is to ensure that a student learns basics of potential flow of fluids and water wave theory. Same will be used in the research of water waves.

Unit I **16 Hours**
Real fluids and ideal fluids, Velocity of fluid at a point, Streamlines, Path lines, Streak lines, Velocity potential, Vorticity vector, Local and particle rate of change, Equation of continuity, Irrigational and rotational motion, Acceleration of fluid, Conditions at rigid boundary.

Unit II **14 Hours**
Euler's equation of motion, Bernoulli's equation and their applications, Potential theorems, Axially symmetric flows, Impulsive motion, Kelvin's Theorem of circulation, Equation of vorticity.

Unit III **16 Hours**
Equations of Motion. Two dimensional flow. Navier-Stokes equation of motion.Velocity potential and Laplace equation. Simple irrotational flows. Separation of variables for an axisymmetric flow. Bernoulli equation for unsteady irrotational flow. Deep water wave. Shallow water wave.

Unit IV **14 Hours**
Theory of surface wave, Finite amplitude wave, One dimensional tidal dynamics, Linear and non-linear diffraction theory, Permutation methods. Water wave interaction with submerged spherical structures and floating cylinders, Solitary

waves, Cnoidal wave, Schrodinger equation.

Suggested Readings:

1. G. K. Batchelor, *An Introduction to Fluid Dynamics*, Cambridge Mathematical Library, 2005.
2. O. M. Phillips, *The Dynamics of Upper Ocean*, Cambridge University Press, 1977.
3. J. J. Stoker, *Water Waves,* Wiley- Blackwell, 1992.

Course Title: Topology
Course Code: PHDMS.717
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes: The course is an introductory course on point-set topology. It is designed in such a way that the students will have a working knowledge in general topology and be able to understand more advanced topics like Algebraic Topology, Differential Topology, Riemannian Geometry and allied areas.

Unit-I **16 Hours**

Topological spaces: Open sets, Closed sets, Neighborhoods, Bases, Sub bases, Limit points, Closures, Interiors, Continuous functions, Homeomorphisms. Examples of topological spaces: Subspace topology, Product topology, Metric topology, Topological manifolds. Quotient Topology: Construction of cylinder, Cone, Mobius band and Torus.

Unit-II **15 Hours**

Connected spaces, Connected subspaces of the real line, Components and path components, Local connectedness. Compact spaces, Sequentially compact spaces, Heine-Borel theorem, Compact subspaces of the real line, Limit point compactness, Local–compactness and one point compactification.

Unit-III **15 Hours**

The Countability axioms: Separable spaces, Lindelöf spaces. Separation axioms: Hausdorff spaces, Regularity, Complete regularity, Normality, Urysohn lemma, Urysohn metrization theorem, Tietze extension theorem and Tychnoff theorem.

Unit-IV **14 Hours**

Covering spaces, Local finiteness, Refinement, The Nagata-Smirnov metrization theorem, Paracompactness, Partition of unity, The Smirnov metrization theorem.

Suggested Readings:

1. M. A. Armstrong, *Basic Topology*, Paperback Edition, Springer, 2004.
2. James Dugundji, *Topology,* Universal Book Stall, New Delhi, 1990.
3. J. L. Kelley, *General Topology*, GTM, First Edition, Springer, 1975.

4. S. Kumaresan, *Topology of Metric Spaces,* second edition, Narosa Publishing House New Delhi, 2015.
5. J. R. Munkres, *Topology,* Second Edition, Pearson India Education services Pvt. Ltd., 2015.
6. G. F. Simmons, *Introduction to Topology & Modern Analysis*, McGraw Hill, Auckland, 1963.

Course Title: Differential Geometry
Course Code: PHDMS.718
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes: To introduce students to the local and global theory of curves and
surfaces so that they can embark on further studies and research in topics like Differential Topology, Riemannian Geometry and allied areas.

Unit-I **15 Hours**

Curves in plane and space: Parameterized curves, Tangent vector, Arc length, Reparametrization, Regular curves, Curvature and torsion of smooth curves, Frenet-Serret formulae, Arbitrary speed curves, Frenet approximation of a space curve. Osculating plane, Osculating circle, Osculating sphere, Involutes and evolutes, Bertrand curves, Spherical indicatrices, Helices, Fundamental theorem of space curves.

Unit-II **15 Hours**

Isomeries of $\mathbb{R}^3$, Congruence of curves. Surfaces in $\mathbb{R}^3$**:** Definition and examples, Smooth surfaces, Tangent, Normal and orientability. Examples of surfaces: Generalized cylinder and generalized cone, Ruled surfaces, Surface of revolution and Quadric surfaces. First fundamental form, Isometries of surfaces, Conformal mapping of surfaces, Surface area, Equi-areal maps and theorem of Archemedes,

Unit-III **15 Hours**

Second fundamental form, Curvature of curves on a surface, Normal and principal curvatures, Meusnier's theorem, Euler's theorem, Weingarten equations and Weingarten matrix, Geometric interpretation of principal curvatures, Umbilical points. Gaussian and mean curvature, Pseudo sphere, Flat surfaces, Surfaces of constant mean curvature, Gaussian curvature of compact surfaces, Gauss map and its properties.

Unit-IV **15 Hours**

Geodesics: Definition and basic properties, Geodesic equations, Geodesics on a surfaces of revolution, Clairaut's theorem, Geodesics as shortest paths, Geodesic coordinates, Gauss Theorema Egregium, Gauss equations, Codazzi-Mainardi equations, Compact surfaces of constant Gaussian curvature.

Suggested Readings:

1. C. Bär, *Elementary Differential Geometry*, Cambridge University Press, 2001.
2. M. P. Do Carmo, *Differential Geometry of Curves and Surfaces*, Revised and Updated Second Edition, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 2016.
3. A. Gray, E. Abbena, and S. Salamon, *Modern Differential Geometry of Curves and Surfaces with Mathematica*, Third edition, CRC Press, 2006.
4. R. S. Millman & G. D. Parkar, *Elements of Differential Geometry*, Englewood Cliffs, N.J. : Prentice Hall, 1977.
5. B. O' Neill, *Elementary Differential Geometry*, Revised Second Edition, Academic Press, 2006.
6. A. Pressley, *Elementary Differential Geometry*, Second Edition, Undergraduate Mathematics Series, Springer-Verlag London Ltd., 2010.
7. T. J. Willmore, *An Introduction to Differential Geometry*, First Edition, Dover Publications, Inc., Mineola, New York, 2012.

Course Title: Differential Topology
Course Code: PHDMS.719
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes: To introduce students to the basics of Differential Topology so that they are able to appreciate better the topics covered in allied courses like Algebraic Topology, Riemannian geometry and Riemann-Finsler geometry as well as be adequately prepared for pursuing research in these topics.

UNIT-I **15 Hours**

Topological manifolds, Charts, Atlases, Smooth manifolds, Examples of smooth manifolds, Manifolds with boundary, Smooth functions on a manifold, Smooth maps between manifolds, Diffeomorphism, Smoothness in terms of components, Examples of smooth maps, Partial derivatives, and the Inverse function theorem.

UNIT-II **15 Hours**

Tangent space and tangent bundle, The Differential of a map, Chain rule, Bases for the tangent space at a point, Curves in a manifold, Submersions, Immersions and embeddings, Smooth covering maps, Critical and regular points, Submanifolds, Rank of a smooth map, Submersion and immersion theorems, Bump functions and partition of unity, Sard's theorem, The Whitney embedding theorem(statement only).

UNIT-III **15 Hours**

Vector fields and Lie bracket. Topological groups, Lie groups: Definition and examples, The product of two Lie groups, Lie subgroups, One parameter subgroups and exponential map, Homomorphism and isomorphism in Lie groups, Lie transformation groups, The tangent space and left invariant vector fields of a Lie group.

UNIT-IV **15 Hours**

Tensor algebra, Differential forms, Cotangent spaces, Pullback of l-forms, k-forms, Exterior product, Differential forms on a circle, Exterior derivative, Exterior algebra and Lie derivative, Global formulas for the Lie and exterior derivatives.

Suggested Readings:

1. W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, 2nd edition, Academic Press, New York, 2003.
2. S. S. Chern, W. H. Chen and K. S. Lam, *Lectures on Differential Geometry*, World Scientific Publishing Co. Pvt. Ltd., 2000.
3. L. Conlon, *Differentiable Manifolds*, 2nd edition, Birkhauser Boston, Cambridge, MA, 2001.
4. N. J. Hicks, *Notes of Differential Geometry*, D. Van Nostrand Reinhold Company, New York, 1965.
5. S. Kumaresan, *A Course in Differential Geometry and Lie Groups (Texts and Readings in Mathematics),* Hindustan Book Agency, 2002.
6. J. M. Lee, *Introduction to Smooth Manifolds*, GTM, Vol. 218, Springer, New York, 2003.
7. L. W. Tu, *An Introduction to Manifolds*, Second edition, Springer, 2011.
8. F. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, Springer, New York, 1983.

Course Title: Algebraic Topology
Course Code: PHDMS.720
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning outcomes: The objective of this course is to introduce the student's concept in Algebraic topology so that they can pursue research in this field and its allied areas.

Unit-I **14 Hours**

The Fundamental group: Homotopy of paths, Homotopy classes, The Fundamental group, Change of base point, Topological invariance, Covering spaces, The Fundamental group of the circle.

Unit-II **14 Hours**

Retractions and fixed points, No Retraction Theorem, The Fundamental theorem of Algebra, The Borsuk-Ulam theorem, The Bisection theorem, Deformation Retracts and Homotopy type, Homotopy invariance.

Unit-III **16 Hours**

Direct sums of Abelian Groups, Free products of groups, Uniqueness of free products, Least normal subgroup, Free groups, Generators and relations, The Seifert-Van Kampen theorem, The Fundamental group of a wedge of circles.

Unit-IV **16 Hours**

Classification of covering spaces: Equivalence of covering spaces, The general lifting lemma, The universal covering space, Covering transformation, Existence of covering spaces.

Suggested Readings:

1. M. A. Armstrong, *Basic Topology,* UTM Springer, 2000.
2. S. Deo, *Algebraic Topology: A Primer (Texts and Readings in Mathematics),* Hindustan Book Agency, 2003.
3. M. J. Greenberg and J. R. Harper, *Algebraic Topology:* A First Course, 2nd Edition, Addison-Wesley Publishing Co, 1997.
4. A. Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.
5. W. S. Massey, *A Basic Course in Algebraic Topology,* SPRINGER (SIE), 2007.
6. J. R. Munkres, *Elements of Algebraic Topology*, Perseus Books, 1995.
7. J. J. Rotman, *An Introduction to Algebraic Topology*, *Text in Mathematics, No. 119*, Springer, New York, 2004.
8. E. H. Spanier, *Algebraic Topology (2nd edition*), Springer-Verlag, New York, 2000.

Course Title: Riemannian Geometry
Course Code: PHDMS.721
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning outcomes: The objective of the course is to introduce students to the basic concepts of Riemannian geometry to prepare them for further studies and research in Riemannian geometry, Finsler geometry, Mathematical Physics and their applications in allied areas.

Unit-I **16 Hours**

Review of differentiable manifolds and vector fields, Covariant differentiation of vector fields and affine connection, Riemannian metric, Riemannian manifolds, Riemannian connection, Fundamental theorem of Riemannian geometry via Koszul's formula.

Unit-II **14 Hours**

Tensors and tensor fields (Riemannian metric as the most significant example), Tensorial property, Covariant differentiation of tensor fields, Riemann curvature tensor, Ricci tensor, Sectional, Ricci and scalar curvatures, Isometries, Notion of covering spaces, Pull-back metrics via diffeomorphisms.

Unit-III **16 Hours**

Covariant differentiation of a vector field along a curve with specific examples, Arc length and energy of a piecewise smooth curve, Geodesics as length minimizing curves, First variation of arc length, To show that geodesics are critical points of the fixed end point first variation formula, Exponential map, Geodesic completeness, Geodesic normal coordinates, Hopf-Rinow theorem (statement only), Geodesic variations, Jacobi fields and Gauss lemma.

Unit-IV **14 Hours**

Second variation formula, The index form (Jacobi fields as minimizers of the index form), Global differential geometry, Spaces of constant sectional curvature, Bonnet-Myers theorem, Cartan-Hadamard theorem, Cartan's theorems (on determination of metric by curvature).

Suggested Readings:

1. M. Berger, *A Panoramic View of Riemannian Geometry*, Springer; 1st Edition, 2003. Corr. 2nd printing, 2007.
2. W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, 2nd Edition, Academic Press, New York, 2003.
3. B. O' Neill, *Semi-Riemannian Geometry with Applications to Relativity*, Academic Press, New York, 1983.
4. S. S. Chern, W. H. Chen and K. S. Lam, *Lectures on Differential Geometry*, World Scientific Publishing , 2000.
5. M. P. Docarmo, *Riemannian Geometry*, Birkhausker Boston, 1992.
6. S. Kumaresan, *A Course in Differential Geometry and Lie Groups (Texts and Readings in Mathematics),* Hindustan Book Agency, 2002.
7. J. M. Lee, *Riemannian Manifolds: An Introduction to Curvature*, GTM, Springer, 1st Edition, 1997.

Course Title: Riemann-Finsler Geometry
Course Code: PHDMS.722
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes: The objective of this course is to enable the students, concepts of Riemann-Finsler geometry so that they can pursue research in this area.

UNIT I **15 Hours**

Minkowski norms, Euler's theorem, Fundamental inequality and its interpretation, Finsler structures: definitions and conventions, Examples: Minkowski and locally Moinkowski spaces, Riemannian manifolds, Randers spaces, Berwald spaces, Finsler spaces of constant flag curvature. Fundamental metric tensor and Cartan tensor.

UNIT II **15 Hours**

Vector bundle, Nonlinear connection on slit tangent bundle, Chern connection, Structure equations, Horizontal and vertical covariant derivatives, hh-, hv-, vvcurvatures, Bianchi identities and their consequences, Ricci identities, Geodesic spray coefficients, Flag curvature: Definition, example and its predecessor, Schur's lemma.

UNIT III **15 Hours**

Rund's differential equation and its consequence, Criterion for strong convexity, Berwald frame, Moore frame, Geometrical setup on sphere bundle, Cartan scalar(I), Landsberg scalar(J) and Gaussian curvature(K), Riemannian arc length of indicatrix, Gauss Bonnet theorem for Landsberg surfaces.

UNIT IV **15 Hours**

Geodesics: Sprays, shortest paths. Projectively equivalent Finsler metrics, Projectively flat metrics, Parallel vector fields, Parallel translations, Berwald metrics, Landsberg metrics, Distortion and S-curvature, Randers metric of isotropic S-curvature.

Suggested Readings:

1. D. Bao, S. S. Chern, Z. Shen, *An Introduction to Riemann Finsler Geometry, Graduate texts in Mathematics 200*, Springer-Verlag, New York, 2000.
2. S. S. Chern and Z. Shen, *Riemann Finsler Geometry,* Nankai Tracts in Mathematics, Vol. 6. World Scientific Publishing Co. Pvt. Ltd., 2005.
3. Z. Shen, *Lectures on Finsler geometry*, World Scientific Press, 2001.
4. M. Matsumoto, *Foundations of Finsler Geometry and Special Finsler Spaces*, Kaisheisha press, Saikawa, Otsu, 520, Japan, (1986).
5. P. L. Antonelli (ed.), *Handbook of Finsler Geometry*, Kluwer Academic Publishers, Dordrecht, The Netherlands, 2003.
6. Xinyue Cheng and Zhongmin Shen, *Finsler geometry-An Approach via Randers spaces*, First Edition, Springer Berlin Heidelberg, 2012.

Course Title: General Relativity
Course Code: PHDMS.723
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes:

The aim of this course is to make the students learn basic concepts of General theory of Relativity, so as to enable the students to pursue research work in this area, Einstein field equations, Cosmology and allied areas.

Unit-I

Transformation of coordinates, Tensor Algebra, Smooth manifolds: Definition and examples, vector fields, Lie brackets and Lie derivatives. Riemannian metric, parallel transport, covariant derivative, affine connection, Riemannian connection, Geodesics, Riemann curvature tensor and its symmetric properties, Ricci tensor, Bianchi identities, Einstein tensor.

Unit-II
Postulates of Special Theory of Relativity, Lorentz Transformation and consequences, Minkowski Diagram, Four dimensional spacetime continuum, Four vector formulation. Transition of Special to General Theory of Relativity, Principle of covariance, Equivalence Principle and consequences, Stress Energy Tensor, Einstein Field Equations and Newtonian limit.

Unit-III
Spherically symmetric solution to Einstein Field Equation in free space and in matter, Schwarzshild line element. Schwarzschild Black Holes. Equation of Planetary Orbits, Crucial tests of General Theory of Relativity, Advance of Perihelion, Gravitational bending of light and Gravitational Redshift.

Unit-IV
Cosmology: Large scale structure of Universe, Galactic Densities and the darkness of the Night Sky, Galactic Number Counts, Olber's paradox, Cosmological principles, Relativistic Universe and models. Einstein and de-Sitter models of static universe, Dynamical Universe, Comoving time, Red Shifts and Horizons, Friedmann-Robertson-Walker line element, Open and Closed Universe, Hubbles law, Early Universe.

Suggested Readings

1. R. Adler , M. Bazin and M. Schiffer, Introduction to General Relativity, McGraw Hill, 1965.
2. S. Caroll, Spacetime and geometry : an introduction to general relativity, Addison Wesley, 2004.
3. J. B. Hartle, Gravity : an introduction to Einstein's general relativity, Pearson education, 2003.
4. R. D. Inverno, Introducing Einstein's relativity, Oxford university press, 2005.
5. C. W. Misner, K. S. Thorne and J. A. Wheeler, Gravitation, W. H. Freeman and Co. 1973.
6. J. V. Narlikar, General Relativity and Cosmology, Macmillan, 1978.
7. A. S. Ramsey, Newtonian Attraction, Cambridge University Press, 1964.
8. B. F. Schutz, A First Course in General Relativity, Cambridge University Press, 2012.
9. S. Weinberg, Gravitation and cosmology: principles and applications of the general theory of relativity, John wiley and Sons, 2004.

Course Title: Probability Theory
Course Code: PHDMS.724
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning outcomes: The course is designed to equip the students with various probability distributions and to develop greater skills and understanding of Sampling and Estimation.

Unit I **13 Hours**

Probability: Definition of probability-classical, Relative frequency, Statistical and axiomatic approach, Addition theorem, Boole's inequality, Conditional probability and multiplication theorem, Independent events, Mutual and pairwise independence of events, Bayes' theorem and its applications.

Unit II **14 Hours**

Random Variable and Probability Functions: Definition and properties of random variables, Discrete and continuous random variables, Probability mass and density functions, Distribution function. **Concepts of Bivariate Random Variable:** Joint, Marginal and conditional distributions.

Mathematical Expectation: Definition and its properties. Variance, Covariance, Moment generating function- Definitions and their properties.Chebychev's inequality.

Unit III **14 Hours**

Discrete distributions: Uniform, Bernoulli, Binomial, Poisson and geometric distributions with their properties.

Continuous distributions: Uniform, Exponential, Gamma and normal distributions with their properties. Central Limit Theorem (Statement only).

Statistical estimation: Parameter and statistic, Sampling distribution and standard error of estimate. Point and interval estimation, Unbiasedness, Efficiency.

Unit IV **15 Hours**

Sampling Theory: Types of sampling, Errors in sampling, Parameter and statistic, **Tests of Significance:** Null Hypothesis, Alternative Hypothesis, One-tailed, Two-tailed tests. Sampling Attributes: Tests of Significance for single proportion and difference of proportions. Sampling of Variables.

Suggested Readings:

1. A. M. Mood, F. A. Graybill and D. C. Boes, *Introduction to the Theory of Statistics*, Mc Graw Hill Book Company, 1966.
2. J. E. Freund., Mathematical Statistics, Prentice Hall of India.
3. S. C. Gupta and V. K. Kapoor, Fundamentals of Mathematical Statistics, S. Chand Pub., New Delhi.
4. M. Speigel, Probability and Statistics, Schaum Outline Series.
5. S. M. Ross,. Introductory Statistics, 2003.
6. R. V. Hogg and T. T. Craig, Introduction to Mathematical Statistics (4^{th} Edition), Collier-McMillan, 1978.

Course Title: Stochastic Processes and Queuing Theory
Course Code: PHDMS.725
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning outcomes: The course on Stochastic Processes and Queuing Theory, is framed to equip the students with knowledge of terms involved in Stochastic Processes as well as concepts and measures in Queuing Theory.

Unit I **15 Hours**

Review of probability, Random variables and distributions, Generating functions and transforms; Stochastic processes, Discrete and continuous-time Markov chains, Renewal processes,

Unit II **15 Hours**

Brownian motion; Characteristics of queueing systems, Little's formula, Markovian and non-Markovian queueing systems, Embedded Markov chain applications to M/G/1, G/M/1, and related queueing systems,

Unit III **15 Hours**

Queues with vacations, Priority queues, Queues with modulated arrival process, Discrete-time queues and matrix-geometric methods in queues; Networks of queues, Open and closed queueing networks,

Unit IV **15 Hours**

Algorithms to compute the performance metrics; Simulation of queues and queueing networks; Application to manufacturing, Computer and communication systems and networks.

Suggested Readings:

1. L. Kleinrock, *Queueing Systems*, Vol. 1: Theory, 1975, Vol. 2: Computer Applications, 1976, John Wiley and Sons.
2. J. Medhi, *Stochastic Models in Queueing Theory* (2nd Edition),, Academic Press, 2002.
   S. Asmussen, *Applied Probability and Queues* (2nd Edition),, Springer, 2003.
3. D. Gross, and C.Harris, *Fundamentals of Queueing Theory*, 3rd Edition, John Wiley and Sons, 1998.
4. R.B. Cooper, *Introduction to Queueing Theory* (2nd Edition), North-Holland, 1981.
5. R. Nelson, Probability, *Stochastic Processes, and Queueing Theory: The Mathematics of Computer Performance Modelling*, Springer-Verlag, 1995.
6. E. Gelenbe, and G. Pujolle, *Introduction to Queueing Networks* (2nd Edition), John Wiley, 1998.

Course Title: Reliability Theory
Course Code: PHDMS.726
Total Hours: 60

L	T	P	Cr
4	0	0	4

Objectives:
The course on Reliability Theory is framed to equip the students with knowledge of terms involved in reliability theory as well as concepts and measures.

Unit I **15 Hours**
Reliability Concepts and Measures: Components and systems, Coherent systems, Reliability of coherent systems, Cuts and paths, Modular decomposition, Bounds on system reliability, Structural and reliability importance of components.

Unit II **15 Hours**
Life distributions and associated survival, Conditional survival and hazard rate functions. Exponential, Weibull, Gamma life distributions and estimation of their parameters.

Unit III **15 Hours**
Notions of ageing. IFR IFRA, NBU, DMRL, NBUE, and HNBUE classes; their duals and relationships between them. Closures of these classes under formation of coherent systems, convolutions and mixtures.

Unit IV **15 Hours**
Partial orderings: Convex, star, stochastic, failure rate and mean-residual life orderings. Univariate shock models and life distributions arising out of them. Maintenance and replacement policies, Availability of repairable systems.

Suggested Readings:

1. R. E. Barlow and F. Proschan, *Statistical Theory of Reliability and Life Testing,* Holt, Rinehart and Winston, 1985.
2. J. F. Lawless, *Statistical Models and Methods of Life Time Data*, John Wiley Models, Marcel Dekker, 1982.
3. M. Shaked and J. G. Shanthikumar, *Stochastic Orders & Their Applications*, Academic Press, 1994.

Course Title: Sampling Theory
Course Code: PHDMS.727
Total Hours: 60

L	T	P	Cr
4	0	0	4

Objectives:
The course is designed to equip the students with basic knowledge of different sampling schemes, their mean and variance estimations and also give understanding of non-sampling errors.

Unit I **15 Hours**
Introduction to usual notations used in sampling. Basic finite population sampling techniques: SRSWOR, SRSWR, stratified, systematic and related results on estimation of population mean/ total. Relative precision of different sampling techniques. Allocation problem in stratifiedsampling.

Unit II **15 Hours**
Ratio and regression estimators based on SRSWOR method of sampling. Two-stage samplingwithequal size of first stage units. Double sampling for ratio and regression methods of estimation. Cluster sampling - equal clusters.

Unit III **15 Hours**
PPS WR/WOR methods [cumulative total, Lahiri's schemes] and related estimators of a finite population mean [Thompson-Horwitz, Yates and Grundy estimator, Desraj estimators for a general sample size and Murthy's estimator for a sample of size 2].

Unit IV **15 Hours**
Sampling and Non-sampling error with special reference to non-response problems. National sample surveys organization (NSSO) and role of various statistical organizations in national development.

Suggested Readings:

1. A. Chaudhuri, *Essentials of Survey Sampling*, Prentice Hall of India, 2010.
2. A. Chaudhari and H. Stenger, *Survey Sampling Theory and Methods*, 2nd Edition, Chapman and Hall, 2005.
3. W. G. Cochran, *Sampling Techniques*, John Wiley & Sons, New York, 1977.
4. A. S. Hedayat and B. K. Sinha, *Design and Inference in Finite Population Sampling*, Wiley, New York, 1991.
5. P. S. Levy and S. Lemeshow, *Sampling of Populations-Methods and Applications*, Wiley, 2008.
6. S. K. Thompson, *Sampling*, John Wiley and Sons, New York, 2002.
7. P. Mukhopadyay, *Theory and Methods of Survey Sampling*, Prentice Hall of India, 1998.