# Central University of Punjab 


M.Sc. MATHEMATICS

Session: 2021-2023

## Department of Mathematics and Statistics

 School of Basic Sciences
## M.Sc. (Mathematics) Programme

## Graduate Attributes:

Students will be able to develop a broad understanding of recent mathematical theories, tools and techniques. Students will apply different mathematical techniques in various fields and will independently plan and carry out research in pure and applied mathematics. They will compete at regional/national/international level for research/jobs in the area of mathematics.

| Course |  | Course |  | it |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Code |  | Type | L | T | P | ts |
| MAT. 506 | Real Analysis | Core | 3 | 0 | 0 | 3 |
| MAT. 508 | Linear Algebra | Core | 3 | 0 | 0 | 3 |
| MAT. 509 | Ordinary Differential Equation | Core | 3 | 0 | 0 | 3 |
| MAT. 525 | Differential Geometry | Core | 3 | 0 | 0 | 3 |
| MAT. 527 | Differential Geometry <br> (Practical) | Skill based | 0 | 0 | 2 | 1 |
| MAT. 530 | Topology | Core | 3 | 0 | 0 | 3 |
| MAT. 553 | Numerical Analysis | Core | 3 | 0 | 0 | 3 |
| MAT. 554 | Numerical Analysis (Practical) | Skill based | 0 | 0 | 2 | 1 |
| XYZ | Interdisciplinary Elective -(From Other Departments) | IDC | 2 | 0 | 0 | 2 |
| Total |  |  | 20 | 0 | 4 | 22 |

Interdisciplinary courses offered by Department of Mathematics and Statistics (For PG students of other Departments)

| MAT.510 | Basic Mathematics (IDC) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MAT.515 | IDC | 2 | 0 | 0 | 2 |  |
| MAT.529 | Numerical Methods (IDC) |  |  |  |  |  |

## M.Sc. Mathematics (Semester-II)

| Course <br> Code | Course Title | Course <br> Type | Credit Hours |  |  | Course Credits |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | L | T | P |  |
| MAT. 526 | Complex Analysis | Core | 3 | 0 | 0 | 3 |
| MAT. 531 | Abstract Algebra | Core | 3 | 0 | 0 | 3 |
| MAT. 552 | Calculus of Variation and Integral equations | Core | 3 | 0 | 0 | 3 |
| MAT. 555 | Differential Manifolds | Core | 3 | 0 | 0 | 3 |
| MAT. 561 | Partial Differential Equations | Core | 3 | 0 | 0 | 3 |
| MAT. 562 | Measure Theory | Core | 3 | 0 | 0 | 3 |
| MAT. 563 | Partial Differential Equations (Practical) | Skill based | 0 | 0 | 2 | 1 |
| MAT. 511 | Number Theory |  |  |  |  |  |
| MAT. 512 | Mathematical Statistics | Discipline |  |  |  |  |
| MAT. 532 | Mathematical Modeling | Elective |  |  |  |  |
| STA. 511 | Operations Research |  | 3 | 0 | 0 | 3 |
| ABC | Value Added Course (From Other Departments) | VAC <br> (Value <br> Based) | 2 | 0 | 0 | 2 |
| Total |  |  | 23 | 0 | 2 | 24 |

Value added courses offered by Department of Mathematics and Statistics (For PG students of other Departments)

| MAT. 528 | Linear Programming | VAC (Value Based) | 2 | 0 | 0 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MAT. 533 | Multivariable Calculus |  |  |  |  |  |
| MAT. 534 | Mathematical Methods |  |  |  |  |  |


| Course <br> Code | Course Title | Course Type | Credit Hours |  |  | Course Credits |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | L | T | $\mathbf{P}$ |  |
| MAT. 502 | Research Methodology | Compulsory Foundation | 4 | 0 | 0 | 4 |
| MAT. 571 | Functional Analysis | Core | 3 | 0 | 0 | 3 |
| MAT. 556 | Advanced Complex Analysis |  |  |  |  |  |
| MAT. 557 | Advanced Partial Differential Equations | Discipline Elective |  |  |  |  |
| MAT. 558 | Algebraic Topology |  |  |  |  |  |
| MAT. 572 | Riemannian Geometry |  | 3 | 0 | 0 | 3 |
| MAT. 559 | Advanced Algebra |  |  |  |  |  |
| MAT. 560 | Discrete Differential Geometry |  |  |  |  |  |
| MAT. 564 | Category Theory | Elective |  |  |  |  |
| MAT. 565 | Dynamical Systems |  |  |  |  |  |
| MAT. 566 | Relativity Theory |  | 3 | 0 | 0 | 3 |
| MAT. 567 | Review of Mathematical Concepts (DEC) | Compulsory Foundation | 2 | 0 | 0 | 2 |
| MAT. 568 | Basics of Latex (Practical) | Skill based | 0 | 0 | 2 | 1 |
| STA. 563 | Entrepreneurship | Compulsory Foundation | 1 | 0 | 0 | 1 |
| MAT. 600 | Research Proposal | Skill based | 0 | 0 | 8 | 4 |
| Total |  |  | 16 | 0 | 10 | 21 |

## M.Sc. Mathematics (Semester IV)

| Course <br> Code | Course Title | Course | Credit Hours |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Type |  | $\mathbf{T}$ | $\mathbf{P}$ | Course <br> Credits |  |
| MAT.600 | Dissertation | Skill Based | 0 | 0 | 40 | 20 |

## Total Credits for the course:

## Evaluation Criteria for Core, Discipline Elective, Compulsory Foundation, VAC and IDC

A. Internal Assessment: [25 Marks]
B. Mid Semester Test: Based on Subjective Type Questions [25 Marks]
C. End Semester Exam: Based on 70\% Subjective Type Questions and 30\% Objective Type Questions [50 Marks]

## Discipline Enrichment Course:

A. Mid Semester Test: Based on Objective Type Questions [50 Marks]
B. End Semester Exam: Based on Objective Type Questions [50 Marks]

## Entrepreneurship Course:

A. Mid Semester Test: Based on Objective Type Questions [25 Marks]
B. End Semester Exam: Based on Objective Type Questions [25 Marks]

## Dissertation:

A. Third semester (Based on proposal)
a) Dissertation proposal and presentation: Supervisor [50 marks]
b) Dissertation proposal and presentation: HoD and Senior most faculty [50 marks]
B. Fourth semester (Based on Dissertation)
c) Continuous assessment, report, presentations, viva voce: Supervisor [50 marks]
d) Continuous assessment, report, presentations, viva voce: HoD, Senior most faculty, and External expert [50 marks]

## Evaluation Criteria for Practical classes

A. Practical file: [15 Marks]
B. Practical Exam: [75 Marks]
C. Viva-Voce Examination: [10 Marks]

## Instructions regarding MOOC courses:

MOOCs May be taken up 40\% of the total credits (excluding dissertation credits). MOOC may be taken in lieu of any course but content of that course should match a minimum $70 \%$. Mapping is to be done by the respective department and students may be informed accordingly.

## Instructions for Dissertation work:

Students will have an option to carry out dissertation work in industry, national institutes or Universities in the top 100 NIRF ranking. Group dissertation may be opted, with a group consisting of a maximum of four students. These students may work using single approach or multidisciplinary approach. Research projects can be taken up in collaboration with industry or in a group from within the discipline or across the discipline.

## M. Sc. Mathematics

## Semester-I

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credits |
| :--- | :--- | :--- | :--- |
| $\mathbf{3}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{3}$ |

Course Code: MAT. 506
Total Lectures: 45

## Learning outcomes:

The students will be able to

- Apply the knowledge of set theory and metric spaces with properties.
- Illustrate various properties of compact sets and connected sets.
- Explain concepts of convergent sequences and continuity in metric spaces.
- Apply the knowledge of Riemann Stieltjes Integrals.


## Unit-I

## 12 Hours

Metric spaces: Definition and examples, Open and closed sets, Compact sets, Elementary properties of compact sets, k- cells, Compactness of k-cells, Compact subsets of Euclidean space $\mathfrak{R}^{k}$, Bolzano Weierstrass theorem, Heine Borel theorem, Perfect sets, Cantor set, Separated sets, Connected sets in a metric space, Connected subsets of real line.

Activity: Students will solve some problems which will be based on concepts of compact sets and connected sets

## Unit-II

## 11 Hours

Sequences in Metric spaces: Convergent sequences, Subsequences, Cauchy sequences, Complete metric space with examples, Cantor's intersection theorem (Statement only), Category of a set and Baire's category theorem. Banach contraction principle.

Activity: Students will solve some problems which will be based on application of sequences, category theorem and Banach contraction theorem.

## Unit-III

12 Hours

Continuity: Limits of functions (in Metric spaces), Continuous functions, Continuity and compactness, Continuity and connectedness, Discontinuities, Uniform continuity.

Riemann Stieltje's Integral: Definition and existence of Riemann Stieltje's integral, Properties of integral. Integration and Differentiation. Fundamental Theorem of Calculus, 1st and 2nd Mean Value Theorems of Riemann Stieltje's integral.

Activity: Students will do examples/exercises related to continuity and its characterizations. Students will explore how Riemann Stieltje's integral is generalization of Riemann integral.

## Unit-IV <br> 10 Hours

Sequences and series of functions: Problem of interchange of limit processes for sequences of functions, Uniform convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation.

Activity: Students will explore how uniform convergence is related to integration and differentiation.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. T. M. Apostol, Mathematical Analysis, Addition -Wesley, USA, 1981.
2. R. G. Bartle, The Elements of Real Analysis, John Willey and Sons, New York, 1976.
3. A. Kumar and S. Kumaresan, A Basic Course in Real Analysis, Narosa, Publishing House, New Delhi, 2014.
4. W. Rudin, Principles of Mathematical Analysis, 3rd Edition, McGraw Hill, Kogakusha, International student Edition, 1976.
5. E. C. Titchmarsh, The Theory of functions, Oxford University Press, Oxford, 2002.

Course Title: Linear Algebra
Course Code: MAT. 508

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credits |
| ---: | :---: | :---: | :---: |
| $\mathbf{3}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{3}$ |

## Learning outcomes:

The students will be able to

- Review the basic notions in linear algebra that are often used in mathematics and other sciences
- Define Vector spaces, Subspaces and related results.
- Define Linear transformations and characteristic polynomials with examples.
- Illustrate various properties of canonical forms.
- Study of Inner product spaces
- Explain concepts of the Gram-Schmidt orthogonalization process.


## Unit I

## 12 Hours

Vector spaces, Subspaces: Definition and Examples, Linear dependence and independence, Basis and dimensions, Coordinates, Linear transformations, Algebra of linear transformations, Isomorphism, Matrix representation of a linear transformation.

Activity: Students will construct different vector spaces like a set of all continuous functions, set of all polynomials. They will define Linear transformation on these spaces.

## Unit II

## 12 Hours

Change of basis, Rank and nullity of a linear transformation. Linear functionals, Dual spaces, Transpose of a linear transformation. Annihilating Polynomials: Characteristic polynomial and minimal polynomial of a linear transformation, Characteristic values and Characteristic vectors of a linear transformation, Cayley Hamilton theorem.

Activity: Students will explore the geometrical and physical meaning of Characteristic values and characteristics vectors.

## Unit III

## 12 Hours

Diagonalizing matrices, Diagonalizing real symmetric matrices, Characteristic polynomials and minimal polynomials of block matrices, Canonical forms: Jordan canonical forms, rational canonical forms. Quotient spaces, Bilinear forms, Symmetric and skew- Symmetric bilinear forms, Sylvester's theorem, quadratic forms, Hermitian forms.

Activity: Students will solve the problems related to applications of canonical forms of matrices.

## Unit IV

09 Hours
Inner product spaces. Norms and distances, Orthonormal basis, Orthogonality, Schwarz inequality, The Gram-Schmidt orthogonalization process.Orthogonal and positive definite matrices.

Activity: Students will explore the application of defining norm and the inner product on vector spaces.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. J. Gilbert and L. Gilbert, Linear Algebra and Matrix Theory, Cengage Learning, 2004.
2. K. Hoffman and R. Kunze: Linear Algebra, $2^{\text {nd }}$ Edition, Pearson Education (Asia) Pvt. Ltd/ Prentice Hall of India, 2004.
3. V. Bist and V. Sahai, Linear Algebra, Narosa, Delhi, 2002.
4. S Lang, Linear Algebra, Undergraduate texts in mathematics, Springer, 1989.

Course Title: Ordinary Differential Equations
Course Code: MAT. 509

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credits |
| ---: | :---: | :---: | :---: |
| 3 | 0 | 0 | 3 |

Total Hours: 45

## Learning outcomes:

The students will be able to

- Define Initial, Boundary value problems and related results.
- Review the basic concepts of ordinary differential equations.
- Study of Stability for Linear systems.
- Explain concepts of the Sturm-Liouville boundary value problem.


## Unit-I

12 Hours
Existence and Uniqueness of Initial Value Problems: Picard's and Peano's Theorems, Gronwall's inequality, continuation of solutions and maximal interval of existence, continuous dependence.

Activity: Students will try to find the applications of existence and uniqueness theorem of initial value problems

## Unit-II

 11
## Hours

Higher Order Linear Equations and linear Systems: fundamental solutions, Wronskian, variation of constants, matrix exponential solution, behaviour of solutions.

Activity: Students will explore the use of Wronskian for checking the behaviour of solutions higher order linear equations and linear Systems

## Unit-III

 12Hours
Boundary Value Problems: Green's function and its applications to boundary value problems, Sturm-Liouville boundary value problem, Eigen values and Eigen functions.

Activity: Students will solve some boundary value problems using the Green's function.
$\begin{array}{ll}\text { Unit-IV } & 10 \\ \text { Hours } & \end{array}$
Two Dimensional Autonomous Systems and Phase Space Analysis: critical points, proper and improper nodes, spiral points and saddle points.

Activity: Students will check the behavior of some dynamical systems by finding their critical points and checking their stability.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. E. A. Coddington and N. Levinson, Theory of ordinary differential equations. McGraw-Hill Book Company, Inc., New York-TorontoLondon, 1955.
2. E. B. Williams and C. DiPrima Richard, Elementary Differential Equations and Boundary Value Problems, 8 ${ }^{\text {th }}$ Edition, John Wiley and Sons, New York, 2005.
3. G. F. Simmons and S. G. Krantz, Differential Equations; Theory, Techniques and Practice, Tata McGraw Hills, 2007.
4. L. Perko, Differential Equations and Dynamical Systems, Springer, 2001.
5. M. Hirsch, S. Smale and R. Deveney, Differential Equations, Dynamical Systems and Introduction to Chaos, Academic Press, 2004
6. S. L. Ross, Differential Equations, 3rd Edition, Wiley, 1984.
7. M. Rama Mohana Rao, Ordinary Differential Equations: Theory and Applications. Affiliated East-West Press Pvt. Ltd., New Delhi, 1980.


Course Title: Differential Geometry
Course Code: MAT. 525

## Total Hours: 45

## Learning outcomes:

The students will be able to

- Learn the basic concepts of plane and space curves.
- Understand the theory of surfaces in $\square^{3}$.
- Define the first and second fundamental forms.
- Illustrate various properties of curvature.
- Explain the theory of geodesics and relation between geometry and topology.


## Unit-I

11 Hours
Curves in plane and space: Parameterized curves, Tangent vector, Arc length, Reparametrization, Regular curves, Curvature and torsion of smooth curves, Frenet-Serret formulae, Arbitrary speed curves, Frenet approximation of a space
curve, Isometries of $R^{3}$, The Tangent Map of an Isometry, Orientation, Congruence of curves.

Activity: To find curvature and torsion of some important plane and space curves and study global properties of curves.

## Unit-II

## 11 Hours

Surfaces in $R^{3}$ : Definition and examples, Smooth surfaces, Smooth maps, Tangents and derivatives, Normal and orientability. Examples of surfaces: Level surfaces, Generalized cylinder and generalized cone, Ruled surfaces, Surface of revolution, Compact surfaces. First fundamental form, Isometries of surfaces, Conformal mapping of surfaces, Equiareal maps and theorem of Archimedes.

Activity: To find the first fundamental form and surface area of some important surfaces.

## Unit-III

## 12 Hours

Second fundamental form, Gauss and Weingarten maps, Normal and geodesic curvatures, Meusnier's theorem, Parallel transport and covariant derivative, Gaussian and mean curvatures, Principal curvatures, Euler's theorem, Surfaces of constant Gaussian curvature, Flat surfaces, Surfaces of constant mean curvature, Gaussian curvature of compact surfaces.

Activity: To find a second fundamental form, principal curvatures and Gaussian curvature of some important surfaces.

## Unit-IV

## 11 Hours

Geodesics: Definition and basic properties, Geodesic equations, Geodesics on a surfaces of revolution, Clairaut's theorem, Geodesics as shortest paths, Geodesic coordinates, Gauss and Codazzi-Mainardi equations,Gauss' Remarkable Theorem, Compact surfaces of constant Gaussian curvature.

Activity: To find geodesics, geodesic equations and compact surfaces of constant Gaussian curvature.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/ Self-learning.

## Suggested Readings:

1. C. Baer, Elementary Differential Geometry, Cambridge University Press, 2001.
2. M. P. Do Carmo, Differential Geometry of Curves and Surfaces, Revised and Updated Second Edition, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 2016.
3. A. Gray, E. Abbena, and S. Salamon, Modern Differential Geometry of Curves and Surfaces with Mathematica, Third edition, CRC Press, 2006.
4. R. S. Millman \& G. D. Parkar, Elements of Differential Geometry, Englewood Cliffs, N.J. : Prentice Hall, 1977.
5. B. O' Neill, Elementary Differential Geometry, Revised Second Edition, Academic Press, 2006.
6. A. Pressley, Elementary Differential Geometry, Second Edition, Undergraduate Mathematics Series, Springer-Verlag London Ltd., 2010.
7. T. J. Willmore, An Introduction to Differential Geometry, First Edition, Dover Publications, Inc., Mineola, New York, 2012.

Course Title: Differential Geometry (Practical)

Course Code: MAT. 527


Total Hours: $\mathbf{3 0}$

## Laboratory work:

Students will use software MATHEMATICA/MATLAB for performing following activities

1. Plotting of plane curves. Computing of length and curvature of plane curves.
2. To determine velocity, speed and acceleration of parameterized curves.
3. Plotting of some special curves and their curvature, plotting of level curves.
4. To define and construct evolutes, involutes and parallel curves.
5. Graph of unit speed plane curve with assigned curvature function.
6. Translating, rotating and reflection of curves.
7. Constructing tangent, normal and binormal vectors and visualizing resulting Frenet frame for a space curve.
8. Plotting and computing curvature and torsion of space curves.
9. Plotting of space curves with assigned curvature and torsion.
10. Investigation of surface patches and associated normal vectors.
11. Visualization of nonorientable surfaces and the Gauss map.
12. Computation of the first fundamental form of various surfaces.
13. Computing the shape operator and various curvatures.
14. Modeling of surfaces formed from straight lines defined in one way or another by a space curve.
15. Construction of surfaces of revolution, obtained by starting from a plane curve.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Selflearning.

## Suggested Readings:

1. C. Baer, Elementary Differential Geometry, Cambridge University Press, 2001.
2. M. P. Do Carmo, Differential Geometry of Curves and Surfaces, Revised and Updated Second Edition, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 2016.
3. A. Gray, E. Abbena, and S. Salamon, Modern Differential Geometry of Curves and Surfaces with Mathematica, Third edition, CRC Press, 2006.
4. R. S. Millman \& G. D. Parkar, Elements of Differential Geometry, Englewood Cliffs, N.J. : Prentice Hall, 1977.


| $\mathbf{3}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{3}$ |
| :--- | :--- | :--- | :--- |

Course Title: Topology
Course Code: MAT. 530
Total Hours: 45

## Learning outcomes:

The students will be able to

- Describe Topological spaces with examples and related concepts in detail.
- Explain continuous functions in topology and its characterizations.
- Understand various topological properties with examples.
- Discuss various separation axioms with their usage to prove many important results in topology.
- Understand important theorems like Urysohn metrization theorem and Tychnoff theorem.
- Understand more advanced topics like Algebraic Topology, Differential Topology, Riemannian geometry and allied areas


## Unit-I

12 Hours
Topological spaces: Open sets, Closed sets, Neighbourhoods, Bases, Sub bases, Limit points, Closures, Interiors, Continuous functions, Homeomorphisms. Examples of topological spaces: Subspace topology, Product topology, Metric topology.

Activity: Students will work with some new topological spaces and continuous functions defined on them. They will try to find spaces which are Homeomorphic or not.

## Unit-II

## 12 Hours

Quotient Topology: Construction of cylinder, Cone, Mobius band and Torus. Connected spaces, Connected subspaces of the real line, Components and path components, Local connectedness.

Activity: Students will work on how various topological surfaces are constructed. They will also work on various examples of connected and disconnected spaces and its applications to detect when two topological spaces are not homeomorphic.

Compact spaces, Sequentially compact spaces, Heine-Borel theorem, Compact subspaces of the real line, Limit point compactness, Localcompactness and one point compactification.

Activity: Students will explore other important topological properties with examples and applications which will again help to determine nonhomeomorphic topological spaces.

## Unit-IV

## 10 Hours

Separation axioms: Hausdorff spaces, Regularity, Complete regularity, Normality, Urysohn lemma, Urysohn Metrization Theorem, Tietze Extension theorem and Tychnoff theorem (Statements only).

Activity: Students will try to find applications of different separation axioms along with its importance to find new results in topology.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. M. A. Armstrong, Basic Topology, Paperback Edition, Springer, 2004.
2. James Dugundji, Topology, Universal Book Stall, New Delhi, 1990.
3. J. L. Kelley, General Topology, GTM, First Edition, Springer, 1975.
4. S. Kumaresan, Topology of Metric Spaces, second edition, Narosa Publishing House New Delhi, 2015.
5. J. R. Munkres, Topology, Second Edition, Pearson India Education services Pvt. Ltd., 2015.
6. G. F. Simmons, Introduction to Topology \& Modern Analysis, McGraw Hill, Auckland, 1963.

Course Title: Numerical Analysis
Course Code: MAT. 553

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credits |
| ---: | ---: | ---: | :---: |
| $\mathbf{3}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{3}$ |

## Learning outcomes:

The students will be able to

- Review the basic concepts of various numerical techniques for a variety of mathematical problems occurring in science and engineering.
- Explain the basic concept of errors.
- Review the numerical techniques for interpolation and approximations with examples.
- Explain the concept of numerical integration and solutions of differential equations.


## Unit-I

## 11 Hours

Error Analysis: Definition and sources of errors, Propagation of errors, Sensitivity and conditioning, Stability and accuracy, Floating-point arithmetic and rounding errors. Numerical Solutions of Algebraic Equations: Bisection method. Fixed-point iteration, Newton Raphson's method, Secant method, Convergence and order of convergence
Activity: Students will explore the use of these methods in solving some real life problems.

## Unit-II

## 12 Hours

Linear Systems of Equations: Gauss elimination and Gauss-Jordan methods, Jacobi and Gauss- Seidel iteration methods.

Polynomial Interpolation: Interpolating polynomial, Lagrange and Newton divided difference interpolation, Error in interpolation, Finite difference formulas, Hermite Interpolation.

Activity: Students will make some programmes for implementing these methods using some computer software.

## Unit-III

## 11 Hours

Numerical Differentiation and Integration: Numerical differentiation with finite differences, Trapezoidal rule, Simpson's $1 / 3$ - rule, Simpson's 3/8 rule, Error estimates for Trapezoidal rule and Simpson's rule, Gauss quadrature formulas.

Activity: Students will make some programmes for implementing these methods using some computer software.

Numerical Solution of Ordinary Differential Equations: Solution by Taylor series, Picard method of successive approximations, Euler's method, Modified Euler method, Runge- Kutta methods. Finite difference method for boundary value problems.

Activity: Students will explore the use of these methods in solving some scientific problems.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. K. Atkinson, An Introduction to Numerical Analysis, $2^{\text {nd }}$ Edition, John Wiley \& Sons, 1989.
2. R. L. Burden and J. D. Faires, Numerical Analysis, 9th Edition, Cengage Learning, 2011.
3. C. F. Gerald and P. O. Wheatly,Applied Numerical Analysis, $7^{\text {th }}$ Edition, Pearson LPE, 2009.
4. R. S. Gupta, Elements of Numerical Analysis, $2^{\text {nd }}$ Edition, Cambridge University Press, 2015.
5. M. K. Jain, S.R.K. Iyengar and R.K. Jain, Numerical Methods for Scientific and Engineering Computation, $6^{\text {th }}$ Edition, New Age International, New Delhi, 2015.
S. S. Sastry, Introductory Methods of Numerical Analysis, 4th Edition, PHI,

Course Title: Numerical Analysis (Practical)
Course Code: MAT. 554
Total Hours: 30

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credits |
| ---: | ---: | ---: | :---: |
| $\mathbf{0}$ | 0 | 2 | 1 |

## Learning outcomes:

The students will be able to

- Explain continuous functions in topology and its characterizations.
- Understand the C/C++/MATLAB languages with examples.
- Understand programming in $\mathrm{C} / \mathrm{C}++/ \mathrm{MATLAB}$ for basic numerical methods.

Laboratory Work: Programming exercises on numerical methods using C/C++/MATLAB languages.

1. To detect the interval(s) which contain(s) root of equation $f(x)=0$ and implement bisection method to find the rootof $f(x)=0$ in the detected interval.
2. To compute the root of equation $f(x)=0$ using Secant method.
3. To find the root of equation $f(x)=0$ using Newton-Raphson and fixed point iteration methods.
4. To compute the intermediate value using Newton's forward difference interpolation formula.
5. To apply Lagrange method for a data set.
6. To construct divided difference table for a given data set and hence compute the intermediate values.
7. To solve a linear system of equations using Gauss elimination (without pivoting) method.
8. To solve a linear system of equations using the Gauss-Seidel method.
9. To find the dominant eigenvalues and associated eigenvectors by Rayleigh power method.
10. To integrate a function numerically using trapezoidal and Simpson's rule.
11. To solve the initial value problem using Euler method.
12. To solve the initial value problem using modified Euler's method.
13. To solve the initial value problem using $2^{\text {nd }}$ and $4^{\text {th }}$ order Runge-Kutta methods.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching / Experimentation /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. K. Atkinson, An Introduction to Numerical Analysis, 2 ${ }^{\text {nd }}$ Edition, John Wiley \& Sons, 1989.
2. R. L. Burden and J. D. Faires, Numerical Analysis, 9th Edition, Cengage Learning, 2011.
3. C. F. Gerald and P. O. Wheatly,Applied Numerical Analysis, $7^{\text {th }}$ Edition, Pearson LPE, 2009.
4. R. S. Gupta, Elements of Numerical Analysis, $2^{\text {nd }}$ Edition, Cambridge University Press, 2015.
5. M. K. Jain, S.R.K. Iyengar and R.K. Jain, Numerical Methods for Scientific and Engineering Computation, 6th Edition, New Age International, New Delhi, 2015.
6. S. S. Sastry, Introductory Methods of Numerical Analysis, $4^{\text {th }}$ Edition, PHI, 2012

Course Title: Basic Mathematics (IDC)
Course Code: MAT. 510
Total Hours: $\mathbf{3 0}$

## Learning outcomes:

The students will be able to

- Define sets and functions with related concepts.
- Define the concept of functions and relations.
- Relate the concept of Arithmetic progression and Geometric progression and their sum.
- Explain the description of algebraic properties of complex numbers.
- Explore the theory of Matrices and Determinants.


## Unit-I

8 Hours
Sets: Basic Definitions, subsets, power set, set operations. Ordered pairs, Cartesian product of sets.

Functions and Relations: Definition of relation, domain, co-domain and range of a relation. Binary relations, equivalence relations, partition. Function as a special kind of relation from one set to another. Domain, co-domain and range of a function.Composition, inverse.Real valued function of the real variable, constant, identity, Polynomial, rational, Functions.

Activity: Students will try to find the applications of functions and relations.

## Unit-II

## 7 Hours

Sequence and series, Arithmetic Progression (A.P), Arithmetic Mean (A.M), Geometric Progression (G.P), general term of a G.P, sum of $n$
terms of a G.P. Arithmetic and Geometric series, infinite G.P. and its sum. Geometric mean (G.M), relation between A.M and G.M.

Activity: Students will solve some problems related to these sequences and series.

## Unit-III

8 Hours
Need for complex numbers, especially $\sqrt{ }-1$, to be motivated by inability to solve every Quadratic equation. Brief description of algebraic properties of complex numbers.Argand plane and polar representation of complex numbers, Statement of Fundamental Theorem of Algebra, $\mathrm{n}^{\text {th }}$ roots of unity.

Activity: Students will solve some problems related to the complex number.

## Unit-IV

## 7 Hours

Matrices and types of matrices, Operations on Matrices, Determinants of Matrix and Properties of Determinants, Minors and Cofactor and Adjoint of a square matrix, Singular and non-singularMatrices, Inverse of a Matrix, Eigenvalues and Eigenvectors, Cayley Hamilton theorem.

Activity: Students will solve some problems related to the matrices and determinants of a matrix.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Reading Books:

1. E. Kreyszig, Advanced Engineering Mathematics, 9 ${ }^{\text {th }}$ edition, John Wiley \& Sons, Inc., 2006.
2. E. Kreyszig, Advanced Engineering Mathematics, 9 th edition, John Wiley \& Sons, Inc., 2006.
3. G. B. Thomas and R. L. Finney, Calculus and Analytic Geometry, $11^{\text {th }}$ edition, Pearson India, 2015.
4. P. K. Jain, Mathematics: Text book for class XI, NCERT, 2006.
5. R. K. Jainand S.R.K. Iyengar, Advanced Engineering Mathematics, 8 ${ }^{\text {th }}$ Edition, Narosa Publications, 2002.

Course Title: Vector Analysis (IDC)
Course Code: MAT. 515

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credits |
| ---: | :---: | :---: | :---: |
| $\mathbf{2}$ | $\mathbf{0}$ | $\mathbf{0}$ | 2 |

Total Lectures: $\mathbf{3 0}$

## Learning outcomes:

The students will be able to

- Explain and use vector dot product and cross product.
- Understand the Gradient, Divergence and Curl.
- Understand and use Green's Theorem, Stokes Theorem and Divergence Theorem.
- Understand Tensors and their applications in Mathematics and allied areas.


## Unit-I

08 Hours
Vectors and Scalars, Vector algebra, Unit vectors, Linear Independence and Linear dependence, Vector fields and scalar fields. Dot and cross product of vectors, Reciprocal set of vectors. Vector differentiation: Ordinary derivative, continuity and differentiability, partial derivatives, Serret-Frenet formulas.

Activity: To check linearly independent and linearly dependent set of vectors, exercises on dot and cross product and vector differentiation.

## Unit-II

## 07 Hours

Gradient, Directional derivative, Divergence, Curl. Vector Integration: Ordinary integral, line integrals, surface integrals and volume integrals. Divergence Theorem of Gauss, Stokes Theorem, Green's theorem in plane.

Activity: Exercises on Gradient, Divergence, Curl and Vector integration.

## Unit-III

07 Hours
Transformation of coordinates, orthogonal curvilinear coordinates, arc length and volume elements, Gradient, Divergence and curl in curvilinear coordinates, special orthogonal coordinate systems.

Activity: Exercises on arc-length, volume, Div. and Curl in Curvilinear coordinates.

Contravariant and covariant vectors. Contravariant, Covariant and Mixed Tensors. Tensors of rank greater than two, Tensor fields, Fundamental operations with Tensors, Line element and metric Tensor, Associated Tensors, Christoffel Symbols, Length of a vector, Angle between vectors, Geodesics, Covariant derivative.

Activity: Exercise on Tensors and their applications.
TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching / Experimentation /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. Narayan Shanti, Mittal P.K., A Text Book of Vector Analysis, S Chand \& Company, Paperback edition, 2010.
2. Murray Spiegel, Seymour Lipschutz, Dennis Spellman, VECTOR ANALYSIS: Schaum's Outlines Series, McGraw Hill Education, 2nd Edition Paperback ,2017.
3. Louis Brand, Vector and Tensor Analysis, Dover Publications, Paperback edition, 2020.
4. A. I. Borisenko, Vector and Tensor Analysis with Applications, Dover Publications, Paperback edition, 2003.
5. Robert C. Wrede, Introduction to Vector and Tensor Analysis, Dover Publications, Paperback edition, 1972.

Course Title: Numerical Methods (IDC)

Course Code: MAT. 529


Total Lectures: $\mathbf{3 0}$

## Learning outcomes:

The students will be able to

- Explain the basic concept of errors.
- Review the basic concepts of various numerical techniques for a variety of mathematical problems occurring in science and engineering.
- Review the numerical techniques for interpolation and approximations with examples.
- Explain the concept of numerical solutions of differential equations.


## Unit-I

7 Hours
Error Analysis: Relative error, Truncation error, Roundoff error, Order of approximation, Order of convergence, Propagation.

Activity: Students will explore the use of error analysis and rounding in some daily life problems like measuring.

## Unit-II

## 8 Hours

Roots of Nonlinear Equations: Bisection method, Secant method, Newton Raphson method, Convergence and order of convergence.

Activity: Students will explore the use of these methods in solving some real life problems.

## Unit-III

## 8 Hours

Linear Systems of Equations: Gauss elimination and Gauss-Seidel methods.Interpolation: Lagrange's Method, Newton's polynomials.

Activity: Students will explore the use of these methods in solving some real life problems.

## Unit-IV

## 7 Hours

Solution of Differential Equations: Euler's method, Heun's method, Taylor series method, Runge-Kutta method.

Activity: Students will explore the use of these methods in solving some scientific problems.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Reading:

1. C. F. Gerald and P. O. Wheatly,Applied Numerical Analysis, $7^{\text {th }}$ Edition, Pearson LPE, 2009. Computation, $6^{\text {th }}$ Edition, New Age International, New Delhi, 2015.
2. J. I. Buchaman and P. R. Turner, Numerical Methods and Analysis, Prentice-Hall, 1988.
3. K. Atkinson, An Introduction to Numerical Analysis, $2^{\text {nd }}$ Edition, John Wiley \& Sons, 2012.
4. M. K. Jain, S.R.K. Iyengar and R.K. Jain, Numerical Methods for Scientific and Engineering
5. R. S. Gupta, Elements of Numerical Analysis, $2^{\text {nd }}$ Edition, Cambridge University Press, 2015.
6. S. S. Sastry, Introduction Methods of Numerical Analysis, 4th Edition, Prentice-Hall, 2005.

## M. Sc. Mathematics

## Semester-II

## Course Title: Complex Analysis

Course Code: MAT. 526
Total Lectures: 45

## Learning outcomes:

The students will be able to

- Recall complex number systems and algebra of complex variables.
- Illustrate the concept of analytic function and discuss the necessary and sufficient conditions for a function to be analytic.
- Understand the notion of complex line integral and related results.
- Discuss Mobius transformations and their properties.
- Apply ideas of Complex analysis in allied areas.


## Unit-I

10 Hours
Functions of a complex variable, limit, continuity, uniform continuity, differentiability, analytic function, Cauchy- Riemann equations, harmonic functions and harmonic conjugate.

Activity: Students will make use of Cauchy- Riemann equations to investigate the functions of complex variables which are analytic or not.

## Unit-II

12 Hours

Complex line integral, Cauchy's theorem, Cauchy-Goursat theorem, Cauchy's integral formula and its generalized form, Cauchy's inequality. Poisson's integral formula (Statement only), Morera's theorem. Liouville's theorem. Contour integral, power series, Taylor's and Laurent's series.

Activity: Students will find the applications of important theorems like Cauchy's theorem and Cauchy's integral formula. They will also work on various examples of contour integrals.

## Unit-III

## 12 Hours

Singularities of analytic functions, Fundamental theorem of algebra, zeros of analytic function, poles, residues, residue theorem and its applications to contour integrals. Maximum modulus principle, Schwarz lemma.

Activity: Students will do the examples related to singularities and poles of analytic functions. They will explore the concept of residues and its applications to solve contour integrals.

## Unit-IV

## 11 Hours

Meromorphic functions, the argument principle, Rouche's theorem, Mobius transformations and their properties, definition and examples of conformal mappings.

Activity: Students will explore the concept of Mobius transformations and its applications in different fields.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. H.S. Kasana, Complex Variables:Theory and Applications, $2^{\text {nd }}$ Edition, PHI Learning Pvt. Ltd, 2005.
2. R. V. Churchill \& J. W. Brown, Complex Variables and Applications, 8th Edition, Tata McGraw-Hill, 2014.
3. S. Ponnusamy, Foundations of Complex Analysis, 2 ${ }^{\text {nd }}$ Edition, Narosa Publishing House, 2007.
4. Theodore W. Gamelin, Complex Analysis. UTM, Springer-Verlag 2001.
5. W. Tutschke and H.L. Vasudeva, An Introduction to Complex Analysis, Classical and Modern Approaches, 1 ${ }^{\text {st }}$ Edition,CRC Publications, 2004.
6. Rajendra Kumar Sharma, Sudesh Kumari Shah and Asha Gauri Shankar, Complex Numbers and Theory of Equations, Anthem Press,2011.

## Course Title: Abstract Algebra

Course Code: MAT. 531

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credits |
| ---: | ---: | ---: | :---: |
| $\mathbf{3}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{3}$ |

Total Hours: 45

## Learning outcomes:

The students will be able to

- Review the basic notions in Group theory.
- Review the Ring theory and ideals with examples.
- Illustrate various properties of Polynomial rings.
- Explain Eisenstein's irreducibility criterion and Unique factorization domain.


## Unit I 12 <br> Hours

Group Theory: Review of basic concepts of Groups, Subgroups, Normal subgroups, Quotient groups, Homomorphism, Cyclic groups, Permutation groups, Even and odd permutations, Conjugacy classes of permutations, Alternating groups, Class equations.

Activity: Students will explore the use of group theory like symmetric groups in real life problems.
Unit II 10
Hours

Normal and Subnormal series, Composition series, Solvable groups, Nilpotent groups. Direct products, Fundamental theorem for finite Abelian groups

Activity: Students will solve some problems related to these concepts.

## Unit III

## Hours

Ring theory: Review of rings, Elementary properties of Rings, Zero Divisors, Nilpotent and idempotent elements, Characteristic of rings, Ideals, Ring homomorphism, Maximal and prime ideals, Nilpotent and nil ideals, Zorn's Lemma.

Activity: Students will explore the use of these methods in some real life problems.

## Unit IV 11 <br> Hours

Polynomial rings in many variables, Factorization of polynomials in one variable over a field. Unique factorization Domains. Euclidean and Principal ideal Domains. Gauss lemma, Eisenstein's irreducibility criterion, Unique factorization in $\mathrm{R}[\mathrm{x}]$, where R is a Unique factorization domain.

Activity: Students will explore the use of these concepts in advance algebra.
TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. M. Artin, Algebra, $2^{\text {nd }}$ Edition, Prentice Hall of India, Delhi, 2011.
2. P. B. Bhattacharya, S. K. Jain and S.R Nagpal, Basic Abstract Algebra, Cambridge University Press, New Delhi, 2003.
3. J. A. Gallian, Contemporary Abstract Algebra, Narosa Publishing House, New Delhi, 2008.
4. N. S. Gopalakrishnan, University Algebra, John Wiley \& Sons, 1986.
5. N. Herstein, Topics in Algebra, $2^{\text {nd }}$ Edition, Wiley Eastern Limited, New Delhi, 2006.
6. I. S. Luthar and I. B. S. Passi, Algebra Vol. II: Rings, Narosa Publishing House, 1999.
7. I. B. S. Passi and I. S. Luthar, Algebra Vol. I: Groups, Narosa Publishing House, 1996.
8. S. Surjeet and Q. Zameeruddin, Modern Algebra, $8^{\text {th }}$ Edition, Vikas Publishing House, New Delhi, 2006.
9. Rajendra Kumar Sharma, Sudesh Kumari Shah and Asha Gauri Shankar, Alegebra I: A Basic Course in Algebra, Pearson Education, 2011.

Course Title: Calculus of Variations and Integral Equations

Course Code: MAT. 552
Total Hours: 45

## Learning outcomes:

The students will be able to

- Explain the basic concept of Functional.
- Review the basic concepts of variational methods, for boundary value problems in ODE's \& PDE's.
- Explain the basic concept of isoperimetric problems.
- Explain the basic concept of Volterra and Fredholm Integral Equations.
- Illustrate various properties of Volterra and Fredholm Integral Equations.


## Unit-I

## 12 Hours

Functional, variation of functional and its properties, fundamental lemma of calculus of variation, Euler's-Lagrange equation of single independent and single dependent variable and application. necessary and sufficient conditions for extrema. Brachistochrone problem, functional involving higher order derivatives.

Activity: Exercises depending on unit-I
Unit-II

## 11 Hours

Sturm-Liouville's theorem on extremals, one sided variations, Hamilton's principle, Hamilton's canonical equation of motion, The principle of least action, Langrange's equations from Hamilton's
principle. variational methods, for boundary value problems in ODE's \& PDE's, isoperimetric problems.

Activity: Exercises depending on unit-II
Unit-III 12

## Hours

Volterra equations: Integral equations and algebraic system of linear equations. $\mathrm{L}_{2}$ kernels and functions of Volterra equation.Volterra equations of first and second kind.Volterra integral equation and linear differential equation.

Activity: Exercises depending on unit-III

## Unit-IV 10 <br> Hours

Fredholm Equations: solution by the method of successive approximations. Solution of Fredholm integral equation for degenerate kernel, solution by the successive approximations, Neumann series and resolvent kernel.

Activity: Exercises depending on unit-IV
TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. H. Goldstein, Classical Mechanics, 2nd Edition, Narosa Publishing House, 1980
2. J. L. Synge and B.A. Griffith, Principle of Mechanics, McGraw-Hill Book Company, 1970.
3. M.D. Raisinghania, Integral equations and boundary value problems, 9th Edition, S. Chand Publishing, New Delhi, 2016.
4. R. P. Kanwal, Linear integral equations, Birkhauser, Boston, 1996.
5. Rakesh Kumar and Nagendra Kumar, Differential Equations and Calculus of Variations, CBS Publishers and Distributors Pvt Ltd, 2013.


Course Title: Differentiable Manifolds
Paper Code: MAT. 555
Total Hours: 45

## Learning outcomes:

The students will be able to

- Explain the basic concept of smooth manifolds and smooth functions.
- Review the basic concepts of Submersions, Immersions and embeddings, Smooth covering maps and Bump functions.
- Explain the concepts of Vector fields, Lie brackets and Lie groups.
- Define the differential forms, exterior derivative, exterior algebra and Lie derivative.


## UNIT-I

 12
## Hours

Topological manifolds, Charts, Atlases, Smooth manifolds, Examples of smooth manifolds, Manifolds with boundary, Smooth functions on a manifold, Smooth maps between manifolds, Examples of smooth maps, Diffeomorphism, Smoothness in terms of components, Partial derivatives, and the Inverse function theorem.

Activity: Exercises on charts, atlases and smooth maps.
UNIT-II
11
Hours
Tangent space and tangent bundle, The Differential of a map, Chain rule, Bases for the tangent space at a point, Rank of a smooth map, Submersions, Immersions and embeddings, Critical and regular points, Submersion and immersion theorems, Smooth covering maps, Submanifolds: Embedded submanifolds, Immersed submanifolds, Bump functions and partition of unity.

Activity: Exercises on Rank of smooth map, submersion, immersion and embeddings.

Vector fields and Lie bracket. Topological groups, Lie groups: Definition and examples, The product of two Lie groups, Lie subgroups, One parameter subgroups and exponential map, Homomorphism and isomorphism in Lie groups, Lie transformation groups, The tangent space and Left invariant vector fields of a Lie group.

Activity: Exercises on vector fields, Lie Groups and Lie subgroups.
UNIT-IV 11
Hours
Differential forms, Cotangent spaces, pullback of differential forms, Exterior product, Exterior derivative, Exterior algebra and Lie derivative, Global formulas for the Lie and exterior derivatives.

Activity: Exercises on Exterior product, exterior derivative and Lie derivative.
TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. W. M. Boothby, An Introduction to Differentiable Manifolds and Riemannian Geometry, 2 ${ }^{\text {nd }}$ edition, Academic Press, New York, 2003.
2. S. S. Chern, W. H. Chen and K. S. Lam, Lectures on Differential Geometry, World Scientific Publishing Co. Pvt. Ltd., 2000.
3. L. Conlon, Differentiable Manifolds, $2^{\text {nd }}$ edition, Birkhauser Boston, Cambridge, MA, 2001.
4. N. J. Hicks,Notes of Differential Geometry, D. Van Nostrand Reinhold Company, New York, 1965.
5. S. Kumaresan, A Course in Differential Geometry and Lie Groups (Texts and Readings in Mathematics), Hindustan Book Agency, 2002.
6. J. M. Lee, Introduction to Smooth Manifolds, GTM, Vol. 218, Springer, New York, 2003.
7. W. Tu, An Introduction to Manifolds, Second edition, Springer, 2011.


Course Code: MAT. 561


## Total Hours: 45

## Learning outcomes:

The students will be able to

- Review the basic concepts of Partial differential equations (PDE).
- Explain the methods for solving nonlinear first order PDEs .
- Classify the second order PDEs into Parabolic, Hyperbolic and Elliptic.


## UNIT-I:

## 10 Hours

Cauchy Problems for First Order Hyperbolic Equations: method of characteristics, Integral surface passing through given curve.

Activity: Students will explore the formation of partial differential equations representing some real phenomena.

## UNIT-II:

## 11 Hours

PDEs of second order with variable coefficients: Classification of second order PDEs, Canonical form, Parabolic, Elliptic and Hyperbolic PDEs, Well posed problems, Super imposition principle.

Activity: Students will explore the formation of second order partial differential equations representing some real phenomena.

## UNIT-III: <br> 12 Hours

Initial and Boundary Value Problems: Lagrange-Green's identity and uniqueness by energy methods. Stability theory, energy conservation and dispersion.

Activity: Students will explore how the solutions of problems behave if we change the initial and boundary conditions.

## UNIT-IV:

## 12 Hours

Laplace equation: mean value property, weak and strong maximum principle, Green's function,

Heat equation: initial value problem, fundamental solution, weak and strong maximum principle and uniqueness results.

Wave equation: uniqueness, D'Alembert's method
Activity: Students will derive the Laplace, heat and wave equations.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. L. C. Evans, Partial Differential Equations. Graduate Studies in Mathematics, $2^{\text {nd }}$ Edition, American Mathematical Society, Indian Reprint, 2014.
2. S. J. Farlow, Partial Differential Equations for Scientists and Engineers, Birkhauser, New York, 1993.
3. F. John, Partial Differential Equations, Springer-Verlag, New York, 1982.
4. K, Sankara, Rao, Introduction to Partial Differential Equations, PHI Learning, 2010.
5. Ian N. Sneddon, Elements of Partial Differential Equations, Dover Publications, 2013.
6. E. DiBenedetto, Partial Differential Equations, Birkhaüser, 1995.

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credits |
| :--- | :--- | :--- | :--- |
| $\mathbf{3}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{3}$ |

## Course Title: Measure Theory

## Course Code: MAT. 562

## Total Hours: 45

## Learning outcomes:

The students will be able to

- Explore the concept of algebras, $\sigma$-algebras and borel sets.
- Define Lebesgue outer measure and Lebesgue measure on R with their characterizations.
- Explain measurable functions and their properties.
- Discuss important theorems related to Lebesgue integral.
- Get in-depth understanding of $L^{p}$ spaces.

$$
\text { Unit-I } 12 \text { Hours }
$$

Semi-algebras, Algebras, Monotone class, o -algebras, Measure and outer measures, Outline of extension of measures from algebras to the generated sigma-algebras: Measurable sets; Lebesgue Measure and its properties. Borel sets, Lebesgue outer measure and Lebesgue measure on R, Translation invariance of Lebesgue measure

Activity: Students will find the outer measure of some sets and will apply these concepts in integration.

## Unit-II

Hours
Continuity of measure and Borel-Cantelli Lemma, Existence of a nonmeasurable set, Measurability of Cantor set.

Measurable functions on a measure space and their properties, Borel and Lebesgue measurable functions, Simple functions and their integrals, Littlewood's three principle and Egoroff's Theorem (statements only).

Activity: Students will find some non-measurable sets and measurable functions. They will also try the proof of Littlewood's three principles and Egoroff's Theorem.

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Unit-III11Hours
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Lebesgue integral on $R$ and its properties. Bounded convergence theorem, Fatou's lemma, Lebesgue monotone convergence theorem, Lebesgue dominated convergence theorem, countable additivity and continuity of integration.

Activity: Students will compare the Riemann integral and Lebesgue integral. They will also work on the counter examples where the above theorems fail to hold.

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Unit-IV
1 1
Hours
```

Functions of bounded variations: Jordan's theorem, $L^{p}$ spaces, Young's inequality, Minkowski's and Hölder's inequalities, Riesz-Fischer theorem (statement only).

Activity: Students will write some functions of bounded variations as the difference of two increasing functions.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. G.de Barra, Measure Theory and Integration, Ellis Horwood Limited, England, 2003.
2. G.B. Folland, Real Analysis, $2^{\text {nd }}$ Edition, John Wiley, New York, 1999.
3. P. R. Halmos, Measure Theory, 14th Edition, Springer, New York, 1994.
4. B. Krishna and A. Lahiri, Measure Theory, Hindustan Book Agency, 2006.
5. I. K. Rana, An Introduction to Measure and Integration, $2^{\text {nd }}$ Edition, Narosa Publishing House, New Delhi, 2005.
6. H. L. Royden, Real Analysis, Macmillan, New York, 1988.

| Course Title: Partial Differential Equations |
| :--- |
| (Practical) |

Course Code: MAT. 563

Total Hours: $\mathbf{3 0}$
List of Practicals (using any software)

1. Overview of software like MATLAB, MATEMATICA etc,
2. Solution of Cauchy problem for first order PDE.
3. Finding the characteristics for the first order PDE.
4. Plot the integral surfaces of a given first order PDE with initial data.
5. Solution of wave equation for the associated conditions.
6. Solution of heat equation for the associated conditions.

## Books Recommended

1. Tyn Myint-U and Lokenath Debnath, Linear Partial Differential Equations for Scientists and Engineers, 4th Ed., Springer, Indian reprint, 2006.
2. S.L. Ross, Differential Equations, 3rd Ed., John Wiley and Sons, India, 2004.
3. Martha L Abell, James P Braselton, Differential Equations with MATHEMATICA, 3rd Ed., Elsevier Academic Press, 2004.

## Course Title: Number Theory

Course Code: MAT. 511

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credits |
| :---: | :---: | :---: | :---: |
| $\mathbf{3}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{3}$ |

## Total Hours: 45

## Learning outcomes:

The students will be able to

- elaborate the concept of Divisibility of integers and Congruences.
- Discuss various important theorems in Number theory.
- Develop the knowledge of Number theoretic functions and explore their usage in various important results.
- Explain the representation of an integer as a sum of two or four squares.
- Describe Quadratic residues and quadratic non-residues with their importance.


## Unit-I

## 12 Hours

Divisibility of Integers, Greatest common divisor, Euclidean algorithm. The theorem of arithmetic, Congruences, Residue classes and reduced residue classes.

Activity: Students will work on different examples related to Residue and reduced residue classes with applications.

## Unit-II

## 11 Hours

Chinese remainder theorem, Fermat's little theorem, Wilson's theorem, Euler's theorem. Arithmetic functions $\sigma(n), d(n), \tau(n), \mu(n)$, Order of an integer modulo $n$, primitive roots for primes.

Activity: Students will explore the use of number theoretic functions in various important results.

The theory of indices, Quadratic residues, Product of quadratic residues and quadratic non-residues, Euler's criterion, The Legendre symbol and its properties, Gauss's lemma, Quadratic reciprocity law, Jacobi symbol and its properties.

Activity: Students will explore the concept of Legendre symbol and its applications to find the solution of quadratic congruences.

## Unit-IV

Representation of an integer as a sum of two and four squares. Diophantine equations $\mathrm{ax}+\mathrm{by}=\mathrm{c}, x^{2}+y^{2}=z^{2}$ and its application to $x^{4}+y^{4}=z^{2}$.

Activity: Students will find the integers which can or cannot be written as a sum of two and four squares.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. W. W. Adams and L. J. Goldstein, Introduction to Number Theory, Prentice Hall Inc., 1976.
2. T. M. Apostol, Introduction to Analytic Number Theory, Springer Verlag, 1976.
3. D. M. Burton, Elementary Number Theory, Tata McGraw-Hill, 7th Edition, New Delhi, 2012.
4. H. Davenport, The Higher Arithmetic: An Introduction to the Theory of Numbers, Cambridge University Press; 8 edition, 2008.
5. G. H. Hardy and E. M. Wright, An Introduction to the Theory of Number, Oxford Univ. Press, U.K., 2008.
6. I. Niven, S. Zuckerman, and H. L. Montgomery, Introduction to Number Theory, Wiley Eastern, 1991.

Course Title: Mathematical Statistics
Course Code: MAT. 512


Total Hours: 45

## Learning outcomes:

The students will be able to

- Define the sample space and concept of random variables (discrete and continuous).
- explore the concept of Moment generating function and characteristic functions with examples.
- Illustrate various properties of Discrete and continuous Distributions.
- Explain concepts of sampling distribution and its standard error, Chisquare, t and F distribution.


## Unit I

## 11 Hours

Concept of random variables (discrete and continuous). Distribution Function and its properties, mean and variance. Bivariate random variables and their joint, marginal and conditional p.m.fs. and p.d.fs. Independence of random variables.

Activity: Students will try to work on various examples related to the concepts of p.m.fs. and p.d.fs of random variables.

## Unit II

## 12 Hours

Expectation, Conditional expectation, Moments, Moment generating function and its properties,, Tchebyshey's inequalities, Markov's inequality, Jensen's inequality, Characteristic function and its elementary properties, weak and strong laws of large numbers, Central Limit theorems (i.i.d. case).

Activity: Students will explore the use of expectations and its properties in various concepts of statistics. They will also solve problems based on moment generating functions, characteristic functions and weak \& strong law of large numbers.

## Unit III

11 Hours

Discrete Distributions: Bernoulli, Binomial, Poisson, hyper-geometric, geometric, negative binomial. Continuous Distributions: Uniform, normal, exponential, gamma, Beta.

Activity: Students will explore the use of these discrete and continuous distributions in real life problems.

## Unit IV

## 11 Hours

Chi-square, t and F distributions and their applications. Elementary concepts in testing of statistical hypotheses, Tests of significance: tests based on normal distribution, Chi-square, t and F statistic.

Activity: Students will do the examples applying the tests based on normal distribution, Chi-square, t and F statistic

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. E. J. Dudewicz and S. N. Mishra, Modern Mathematical Statistics, Wiley International Student Edition, 1988.
2. I. Miller and M. Miller, Mathematical Statistics, $6^{\text {th }}$ Edition, Oxford \& IBH Pub., 1999.
3. P. Billingsley, Probability and Measure,4th Edition, John Wiley \& Sons, 2012.
4. P.L. Meyer, Introductory probability and statistical applications, AddisonWesley Publishing Company, Inc., 1972.
5. S. M. Ross, Introduction to Probability Models, $11^{\text {th }}$ Edition, 2014.
6. V. K. Rohtagi and A. K. M. E. Saleh, An Introduction to Probability Theory and Mathematical Statistics, Wiley Eastern, 2010.

## Course Title: Mathematical Modeling <br> Course Code: MAT. 532

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credits |
| :---: | :---: | :---: | :---: |
| $\mathbf{3}$ | $\mathbf{0}$ | $\mathbf{0}$ | 3 |

Total Hours: 45

## Learning outcomes:

The students will be able to

- Understand the mechanics, uses, and limitations of the modeling process
- Clearly explain various methods used to model data
- Apply problem-solving strategies confidently to reach viable solutions of real-world problems


## Unit-I

## 11 Hours

Overview of mathematical modeling and types of mathematical models, Introduction to population dynamics, solution methods of linear difference equations and discrete time model.

Activity: Students will study some epidemic models.

## Unit-II

## 11 Hours

Linear system theory, stability analysis, role of eigenvalues \& vectors and phase diagrams.

Activity: Students will study some autonomous systems and will check their stability.

## Unit-III

## 11 Hours

Single-species population model, Allee effect, Predator-Prey model, LotkaVolterra model and SIR model.

Activity: Students will study some population models with available data.

## Unit-IV

## 12 Hours

Introduction to models in chemical-kinetics, Hopf bifurcation, PoincareBendixson theory and index theory.

Activity: Students will study the bifurcation of some first order differential equation systems.

# TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co 

 Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.
## Suggested Readings:

1. Brauer, F., Driessche, P. V. D. and Wu, J., Mathematical Epidemiology, Springer, 2008.
2. Keshet, L. E., Mathematical Models in Biology, SIAM, 1988.
3. Giordano, Fox, Horton, A First Course in Mathematical Modeling, 5th edition, Cengage, 2013.
4. Clive L. Dym, Principles of Mathematical Modelling, Elsevier Press, Second Edition, 2004.
5. Edward A. Bender, An Introduction to Mathematical Modeling, Dover, 2000.

## Course Title: Operations Research

## Course Code: STA-511



Total Hours: 45

## Learning outcomes:

The students will be able to

- Discuss the concept of convex sets and linear programming problems with formulation.
- Apply different methods to solve linear programming problems.
- explore the concept of Duality theory and Sensitivity analysis.
- Explain transportation problems and assignment problems with their mathematical formulation.
- Apply methods to test the optimality of transportation problems.
- Develop understanding of Queuing and inventory models.


## Unit-I

11 Hours

Mathematical formulation of linear programming problem, Linear Programming and examples, Convex Sets, Hyper plane, Open and Closed half-spaces, Feasible, Basic Feasible and Optimal Solutions, Extreme Point \& graphical methods.

Activity: Students will formulate linear programming problems and find solutions with graphical methods.

## Unit-II <br> 12 Hours

Simplex method, Big-M method, Two phase method, Determination of Optimal solutions, Unrestricted variables. Duality theory, Dual linear Programming Problems, Fundamental properties of dual problems, Complementary slackness, Unbounded solution in Primal. Dual Simplex Algorithm.

Activity: Students will do problems to find solutions through Simplex method, Big-M method and two phase method. They will exercise of dual linear programming problems.

## Unit-III

## 12 Hours

Sensitivity analysis: Discrete changes in the cost vector, requirement vector and coefficient matrix.

The General transportation problem, Duality in transportation problem, Loops in transportation tables, Solution of transportation problem, Test for optimality, Degeneracy, Transportation algorithm (MODI method), Minimization transportation problem.

Activity: Students will do exercises on Sensitivity analysis and transportation problems.

## Unit -IV

## 10 Hours

Assignment Problems: Mathematical formulation of assignment problem, Hungarian method for solving assignment problems, Traveling salesman problem. Sequencing Problem: General assumptions and basic terms used in sequencing. Processing $n$ jobs through 2 machines, Processing $n$ jobs through 3 machines.

Activity: Students will do exercises on Assignment problem and sequencing problem

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. G. Hadley, Linear Programming, Narosa Publishing House, New Delhi, 1987.
2. H. A. Taha, Operations Research - An Introduction, Macmillan Publishing Company Inc., New York, 2006.
3. K. Swarup, P. K. Gupta, and M. Mohan, Operations Research, Sultan Chand \& Sons, New Delhi, 2001.
4. N. S. Kambo, Mathematical Programming Techniques, Affiliated EastWest Press Pvt. Ltd., 1984, Revised Edition, New Delhi, 2005.
5. S. M. Sinha, Mathematical Programming, Theory and Methods, Delhi: Elsevier, 2006.

Course Title: Linear Programming (VAC)
Course Code: MAT. 528
Total Hours: 30

## Learning outcomes:

The students will be able to

- Discuss the linear programming problem with formulation.
- Apply different methods to solve linear programming problems.
- explore the concept of Duality theory and Sensitivity analysis.
- Explain transportation problems and assignment problems with their mathematical formulation.


## Unit-I

 08 HoursFormulation of linear programming problems (LPP).Graphical solution to LPPs. Cases of unique and multiple optimal solutions.
Activity: Students will do formulation of Linear programming problem and find the solutions using graphical method.

## Unit-II

Feasible solution, basic feasible solutions, Optimal solution, Convex sets, Solution of LPP with Simplex methods.

Activity: Students will solve linear programming problems with the simplex method.

## Unit-III

## 06 Hours

The dual problem.Formulation of the dual.Dual Simplex method
Activity: Students will do exercises related to dual linear programming problems.

## Unit-IV

## 08 Hours

Transportation and Assignment Problem: Transportation problems, Formulation of transportation problems, Feasible and optimal solution of transportation problems. Assignment problems.

Activity: Students will do exercises on transportation problems and assignment problems.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. H. A. Taha, Operations Research - An Introduction, Macmillan Publishing Company Inc., New York, 2006.
2. K .Swarup, P. K. Gupta and Man Mohan, Operations Research, Sultan Chand \& Sons, New Delhi, 2001.
3. F. S. Hillier and G. J. Lieberman, Introduction to Operations Research, McGraw-Hill, New York, 2001.

Course Title: Multivariable Calculus (VAC)

Course Code: MAT. 533


Total Hours: $\mathbf{3 0}$

## Learning outcomes:

The students will be able to

- Review the basic concepts of differentiability in $\mathrm{R}^{\mathrm{n}}$.
- Understand the basic concepts of integration in $\mathrm{R}^{\mathrm{n}}$.
- Discuss the Differential forms on $\mathrm{R}^{\mathrm{n}}$.
- Apply the Divergence theorem and Stokes' formula for solving some integrations.
- Understand the notion of the Partitions of Unity.


## Unit-I

7 Hours
Functions in $\mathrm{R}^{\mathrm{n}}$, Differentiability in $\mathrm{R}^{\mathrm{n}}$, directional derivatives, Total derivative, Chain rule, Inverse function theorem, Implicit function theorem.

Activity: Exercises on Total derivative and directional derivative. Applications of Inverse function theorem and Implicit function theorem.

## Unit-II

8 Hours
Integration on $\mathrm{R}^{\mathrm{n}}$, Riemann integral of real-valued functions on Euclidean spaces, Fubini's Theorem, Partitions of Unity and change of variables.

Activity: Exercises on Riemann Integration and Partitions of Unity.

## Unit-III

7 Hours
Differential forms on $\mathrm{R}^{\mathrm{n}}$, closed and exact forms, Poincaré lemma, Classical Green's theorem.

Activity: Exercises on differential forms and applications of Green's theorem.

## Unit-IV

## 8 Hours

The volume element, Divergence theorem and Stokes' formula as applications of the general form of Stokes' theorem.

Activity: Exercises on the applications of Divergence and Stoke's Theorems.

## Suggested Readings:

1. S. R.Ghorpade and B. V. Limaye, A course in Calculus and Real Analysis, Springer, New York, 2006.
2. S. Kumaresan, A Course in Differential Geometry and Lie groups, Hindustan Book Agency, Trim 22, 2002.
3. Walter Rudin, Principles of Mathematical Analysis, Third Edition, McGraw Hill International Editions, Mathematical Studies 1976; Paper-back Indian Edition 2017.
4. M. Spivak, Calculus on Manifolds, W. A. Benjamin, co. 1965.

Course Title: Mathematical Methods (VAC)

Course Code: MAT. 534
Total Hours: 30


## Learning outcomes:

- Students will be able toIntegral Transforms.
- Understand the basic concepts of wavelet transforms.
- Discuss the Differential forms on $\mathrm{R}^{\mathrm{n}}$.
- Understand the method of reduction of IVPs BVPs and eigenvalue problems.
- apply regular and singular perturbation methods.


## Unit-I

## ` 8 Hours

Integral Transforms: General definition of Integral transforms, Kernels, etc. Development of Fourier integral, Fourier transforms - inversion, Illustration on the use of integral transforms, Laplace, Fourier, Hankel and Mellin transforms to solve ODEs and PDEs - typical examples.

## Unit-II

7 Hours

Discrete orthogonality and Discrete Fourier transform. Wavelets with examples, wavelet transforms.

Integral Equations: Definition, Volterra and Fredholm integral equations. Solution by separable kernel, Neumann's series resolvent kernel and transform methods, Convergence for Fredholm and Volterra types.

Activity: Students will write some differential equations in the form of integral equations and then will solve them.

## Unit-III

## 7 Hours

Reduction of IVPs BVPs and eigenvalue problems to integral equations. Hilbert Schmidt theorem, Raleigh Ritz and Galerkin methods.

Activity: Students will use Raleigh Ritz and Galerkin methods to solve some scientific problems.

## Unit-IV <br> 8 Hours

Regular and singular perturbation methods: Parameter and co-ordinate perturbations. Regular perturbation solution of first and second order differential equations involving constant and variable coefficients.

Activity: Students will obtain the regular perturbation solutions of some first and second order differential equations.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. I.N. Sneddon - The use of Integral Transforms, Tata Mc Graw Hill, Publishing Company Ltd, New Delhi, 1974.
2. R.P. Kanwal: Linear integral equations theory and techniques, Academic Press, New York, 1971.
3. C.M. Bender and S.A. Orszag - Advanced mathematical methods for scientists and engineers, McGraw Hill, New York, 1978.

## M. S. Mathematics (Semester-III)

## Course Title: Research Methodology

Course Code: MAT. 502

| $L$ | $T$ | $\mathbf{P}$ | Credits |
| :---: | :---: | :---: | :---: |
| $\mathbf{4}$ | 0 | 0 | 4 |

Total Hours: 45

## Learning outcomes:

The students will be able to

- Explain the various terms like objective, meaning of research, significance research etc. which is used in research.
- Review the basic concepts of literature survey and formulation research problems.
- Basic concepts of the research design.
- Review the basic concepts of the research writing problems.


## Unit-I

## 14 Hours

Introduction: Meaning, Objectives, Characteristics, Significance, and Types of Research; Research Approaches, Research Methods vs. Research Methodology, Research Process, and Criteria of Good Research.

Activity: Students will gain theoretical and practical knowledge of a specific area of research.

## Unit-II

Literature Survey and Review: Meaning of Literature Survey and Review, Sources of Literature, Methods of Literature Review, and Techniques of Writing the Reviewed Literature. Formulating Research Problem: Understanding a Research Problem, Selecting the Research Problem, Steps in Formulation of a Research Problem, Formulation of Research Objectives, and Construction of Hypothesis.

Activity: Students will gain a literature survey and formulate research problems of a specific area of research.

## Unit-III

## 14 Hours

Research Design: Meaning of and Need for Research Design, Characteristics of a Good Research Design, Different Research Designs, Basic Principles of Experimental Designs, Data Collection, Processing, and Interpretation.

Activity: Students will explore the concept of research design and data collection of a research problem of a specific area of research.

## Unit-IV

## 16 Hours

Report Writing: Types of Reports - Technical and Popular Reports, Significance of Report Writing, Different Steps in Writing Report, Art of Writing Research Proposals, Research Papers, Project Reports, and Dissertations/Thesis; Basics of Citation and Bibliography/Reference Preparation Styles; Report Presentation: Oral and Poster Presentations of Research Reports.

Activity: Students will explore the concept of the report writing and report presentation of a research problem.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. Anderson, J. (2001): Thesis and Assignment Writing, $4^{\text {th }}$ ed., Wiley, USA
2. Dawson, Catherine, (2014): Practical Research Methods, New Delhi, UBS Publishers' Distributors.
3. Gray, David E. (2004): Doing Research in the Real World. London, UK: Sage Publications.
4. Kothari, C.R. and G. Garg (2014): Research Methodology: Methods and Techniques, $3^{\text {rd }}$ ed., New Age International Pvt. Ltd. Publisher
5. Kumar, R. (2014): Research Methodology - A Step-By-Step Guide for Beginners, 4th ed., Sage Publications

## Course Title: Functional Analysis

Course Code: MAT. 571


## Total Hours: 45

## Learning outcomes:

The students will be able to

- Describe the basic notion of normed linear spaces and Banach spaces with examples.
- Explain Bounded linear transformations and Dual spaces with related examples
- Discuss three main theorems on Banach spaces
- Understand the concept of Reflexive spaces
- Define inner product spaces and Elaborate Geometry of Hilbert spaces


## Unit-I

## 10 Hours

Fundamentals of Normed Linear Spaces: Normed Spaces, with examples of Function spaces $L^{P}([\mathrm{a}, \mathrm{b}]), \mathrm{C}([\mathrm{a}, \mathrm{b}])$ and $C^{1}([\mathrm{a}, \mathrm{b}])$, Sequence Spaces $l^{p}$, c , $c_{0}, c_{00}$ Banach spaces and examples, finite dimensional normed spaces and subspaces.
Activity: Students will find examples of Banach spaces and normed linear spaces which are not Banach spaces.

## Unit-II

## 11 Hours

Linear operators definition and examples, Bounded linear transformations, Normed linear spaces of bounded linear transformations, Concept of algebraic Dual and algebraic reflexive spaces, Dual spaces with examples

Activity: Students will do results related to bounded linear transformations and Dual spaces.

## Unit-III

Geometry of Hilbert spaces: Inner product spaces and Hilber spaces, Further properties of inner product spaces, orthonormal sets, Approximation and optimization, Projections and Riesz Representation theorem for Hilbert spaces. Bounded Operators on Hilbert spaces: Bounded operators and adjoints; normal, unitary and self adjoint operators.

Activity: Students will do exercises on Hilbert spaces and examples of inner product spaces which are not Hilbert spaces.

## Unit-IV

## 12 Hours

Hahn-Banach theorem for real linear spaces and its consequences, Reflexive spaces, Solvability of linear equations in Banach spaces.

Three Main Theorems on Banach Space: Banach Steinhauns theorem (Uniform boundedness theorem) and some of its consequences, Open mapping and closed graph theorems.

Activity: Students will work on the consequences of Hahn-Banach theorem and Uniform boundedness theorem.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. S. K. Berberian, Introduction to Hilbert Spaces, AMS Chelsea Publishing, Rhode Island, 1996.
2. C. Goffman, and G. Pedrick, First Course in Functional Analysis, Prentice Hall of India, New Delhi, 1983.
3. E. Kreyszig, Introductory Functional Analysis with Application, Willey, 2007.
4. B. V. Limaye, Functional Analysis, New Age International (P) Ltd, New Delhi, 1996.
5. F. K. Riesz, and B. S. Nagy, Functional Analysis, Dover Publications, 1990.
6. A. H. Siddiqui, Functional Analysis, Tata-McGraw Hill, New Delhi, 1987.
7. W. Rudin, Functional Analysis, McGraw Hill Education; 2 edition, 2017.

## Course Title: Advanced Complex Analysis

Course Code: MAT. 556

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credits |
| :---: | :---: | :---: | :---: |
| $\mathbf{3}$ | 0 | 0 | 3 |

## Total Hours: 45

## Learning outcomes:

The students will be able to:

- understand further deeper topics of Complex Analysis
- learn basic topics needed for students to pursue research in pure Mathematics.
- Apply the Weierstrass Zeta function in solving some problems


## Unit-I

## 11 Hours

Harmonic function: definition, relation between a harmonic function and an analytic function, examples, harmonic conjugate of a harmonic function, poisson's integral formula, mean value property, the maximum \& minimum principles for harmonic functions, Dirichlet problem for a disc and uniqueness of its solution, characterization of harmonic functions by mean value property.

Activity: Students will do some exercise on Dirichlet problems.

## Unit-II

11 Hours
Analytic continuation: direct analytic continuation, analytic continuations along arcs, homotopic curves, the monodromy theorem, analytic continuation via reflection. Harneck's principle. Open mapping theorem, normal families, the riemann mapping theorem, Picard's theorem.

Activity: Students will explore the applications of Open mapping theorem, Riemann mapping theorem, Picard's theorem.

## Unit-III

12 Hours

Weierstrass Elliptic functions: periodic functions, simply periodic functions, fundamental period, Jacobi's first and second question, doubly periodic functions, elliptic functions, pair of primitive periods, congruent points, first and second Liouville's theorem, relation between zeros and poles of an elliptic function, definition of Weierstrass elliptic function $(z)$ and their properties, the differential equation satisfied by $(z)$ [i.e., the relation between $(z)$ and ( )], Integral formula for $(z)$, addition theorem and duplication formula for $(z)$.

Acitivity: students will apply the Weierstrass Zeta function in solving some problems

## Unit- IV

## 11 Hours

Weierstrass Zeta function: Weierstrass zeta function and their properties, quasi periodicity of $(z)$, Weierstrass sigma function $(z)$ and their properties, associated sigma functions.

Acitivity: Students will practice the properties of Weierstrass zeta function and Weierstrass sigma function.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. J. B. Conway, Functions of One Complex Variable, 2nd Edition, SpringerVerlag International, USA, 1978.
2. L.V. Ahlfors, Complex Analysis: An Introduction to the Theory of Analytic Functions of One Complex Variable, 3rd Edition, McGraw-Hill, Higher Education, New Delhi, 1979.
3. R. Walter, Real and Complex Analysis, 3rd Edition, McGraw-Hill Book Co., New Delhi, 1986.
4. S. Lang, Complex Analysis, 4 ${ }^{\text {th }}$ Edition, Springer, New York, 2003.
5. S. Ponnusamy, Foundations of Complex Analysis, $2^{\text {nd }}$ Edition, Narosa Publication House, New Delhi, 1995.

## Course Title: Advanced Partial Differential Equations

Course Code: MAT. 557

## Total Hours: 45

## Learning outcomes:

The students will be able to

- use smooth functions for approximations.
- understand the basic concepts of Finite element methods
- study scientific evolution equations.
- learn the conditions for existence of minimizers


## Unit-I

12 Hours

Distribution: Test functions and distributions, examples, operations on distributions, supports and singular supports, convolution, fundamental solutions, fourier transform, Schwartz space, tempered distributions.

Sobolev Spaces: Basic properties, approximation by smooth functions, extension theorems, compactness theorems, dual spaces, functional order spaces, trace spaces, trace theory, inclusion theorem.

Activity: Students will use smooth functions for approximations.

## Unit-II

11 Hours
Weak solutions of elliptic boundary value problems: variational problems, weak formulation of elliptic PDE, regularity, Galerkin method, Maximum principles, eigenvalue problems, Introduction to finite element methods.

Activity: Students will obtain weak solutions of some elliptic boundary value problems.

## Unit-III

## 11 Hours

Evolution Equations: unbounded linear operators, $\mathrm{C}_{0}$ - semigroups, HilleYosida theorem, contraction semigroup on Hilbert spaces, heat equation, wave equation, Schrödinger equation, inhomogeneous equations.

Activity: Students will study scientific evolution equations.

## Unit-IV

## 11 Hours

Calculus of Variations: Euler-Lagrange equation, second variation, existence of minimizers (coactivity, lower semi-continuity, convexity), regularity, constraints (nonlinear eigenvalue problems, variational inequalities, harmonic maps, incompressibility), critical points (mountain pass theorem and applications to elliptic PDE).

Activity: Students will check the conditions for existence of minimizers.
TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. S. Kesavan, Topics in Functional Analysis and Application, WileyEastern, New International, New Delhi, 1999.
2. L. C. Evans, Partial Differential Equations. Graduate Studies in Mathematics, $2^{\text {nd }}$ Edition, American Mathematical Society, Indian Reprint, 2014.
3. K. S. Rao, Introduction to Partial Differential Equation, $2^{\text {nd }}$ Edition, PHI Learning Pvt. Ltd. 2010.
4. T. Amarnath, An Elementary Course in Partial Differential Equations, $2^{\text {nd }}$ Edition, Narosa Publishing House 2012.
5. I. N. Sneddon, Elements of Partial Differential Equations, McGraw-Hill Book Company, New York 1988.

## Course Title: Algebraic Topology <br> Course Code: MAT. 558

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credits |
| ---: | :---: | :---: | :---: |
| $\mathbf{3}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{3}$ |

## Learning outcomes:

The students will be able to

- Explain the fundamental groups, homotopy and covering spaces.
- Understand the Retractions and fixed points, The Borsuk - Ulam and Bisection theorem.
- Understand the free groups, direct sums, and free products.
- Understand the classification of covering spaces.


## Unit-I

## 11 Hours

The Fundamental group: Homotopy of paths, Homotopy classes, The Fundamental group, Change of base point, Topological invariance, Covering spaces, The Fundamental group of the circle.

Activity: To find fundamental groups of some important topological structures.

## Unit-II

## 12 Hours

Retractions and fixed points, No Retraction Theorem, The Fundamental theorem of Algebra, The Borsuk - Ulam theorem, The Bisection theorem, Deformation Retracts and Homotopy type, Homotopy invariance.

Activity: To find applications of above important theorems in terms of examples.

## Unit-III

## 11 Hours

Direct sums of Abelian Groups, Free products of groups, Uniqueness of free products, Least normal subgroups, Free groups, Generators and relations, The Seifert-Van Kampen theorem, The Fundamental group of a wedge of circles.

Activity: Exercises on Abelian groups, free groups and free products.

## Unit-IV

11 Hours
Classification of covering spaces: Equivalence of covering spaces, The general lifting lemma, The universal covering space, Covering transformation, Existence of covering spaces.

Activity: Exercises on covering spaces.

## Suggested Books:

1. James R. Munkres, Elements of Algebraic Topology, Perseus Books (11 December 1995).
2. M. J. Greenberg and J. R. Harper, Algebraic Topology: A First Course (2nd edition), Addison-Wesley Publishing Co, 1997.
3. A. Hatcher, Algebraic Topology, Cambridge University Press, 2002.
4. M. A. Armstrong, Basic Topology, UTM Springer, 2000.
5. E. H. Spanier, Algebraic Topology (2nd edition), Springer-Verlag, New York, 2000.
6. J. J. Rotman, An Introduction to Algebraic Topology, Text in Mathematics, No. 119, Springer, New York, 2004.
7. W. S. Massey, A Basic Course in Algebraic Topology, SPRINGER (SIE), 2007.
8. Satya Deo, Algebraic Topology: A Primer (Texts and Readings in Mathematics), Hindustan Book Agency, 2003.

## Course Title: Riemannian Geometry

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credit <br> $\mathbf{s}$ |
| :--- | :--- | :--- | :--- |
| $\mathbf{3}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{3}$ |

Course Code: MAT. 572
Total Hours: 60

## Learning outcomes:

The students will be able to

- Understand the concepts of differentiable manifolds and covariant differentiation of vector fields
- Explain the theory of Tensors and curvature .
- Discuss the Jacobi fields and Gauss lemma.
- Understand the concept of Global differential geometry and related properties.
- Purse research in allied areas of Differential geometry and Mathematical Physics.


## Unit-I

## 12 Hours

Review of differentiable manifolds and vector fields, Covariant differentiation of vector fields and affine connection, Riemannian metric, Riemannian manifolds, Riemannian connection, Fundamental theorem of Riemannian geometry via Koszul's formula.

Activity: Exercises on covariant differentiation, Reiemannian metric and Riemannian connection.

## Unit-II <br> 11 Hours

Tensors and tensor fields (Riemannian metric as the most significant example), Tensorial property, Covariant differentiation of tensor fields, Riemann curvature tensor, Ricci tensor, Sectional, Ricci and scalar curvatures, Isometries, Notion of covering spaces, Pull-back metrics via diffeomorphisms.

Activity: Exercises on different curvature tensors and isometries.

## Unit-III

## 11 Hours

Covariant differentiation of a vector field along a curve with specific examples, Arc length and energy of a piecewise smooth curve, Geodesics as length minimizing curves, First variation of arc length, To show that geodesics are critical points of the fixed end point first variation formula, Exponential map, Geodesic completeness, Geodesic normal coordinates, Hopf-Rinow theorem (statement only), Geodesic variations, Jacobi fields and Gauss lemma.

Activity: Exercises on arc-length, energy, geodesics and exponential map.

## Unit-IV <br> 11 Hours

Second variation formula, The index form (Jacobi fields as minimizers of the index form), Global differential geometry, Spaces of constant sectional curvature, Bonnet-Myers theorem, Cartan-Hadamard theorem, Cartan's theorems (on determination of metric by curvature).

Activity: Exercises on the application of the above mentioned theorems.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. M. Berger, A Panoramic View of Riemannian Geometry, Springer; $1^{\text {st }}$ Edition, 2003. Corr. 2 ${ }^{\text {nd }}$ printing, 2007.
2. W. M. Boothby, An Introduction to Differentiable Manifolds and Riemannian Geometry, $2^{\text {nd }}$ Edition, Academic Press, New York, 2003.
3. S. S. Chern, W. H. Chen and K. S. Lam, Lectures on Differential Geometry, World Scientific Publishing, 2000.
4. M. P. Docarmo, Riemannian Geometry, Birkhausker Boston, 1992.
5. S. Kumaresan, A Course in Differential Geometry and Lie Groups (Texts and Readings in Mathematics), Hindustan Book Agency, 2002.
6. J. M. Lee, Riemannian Manifolds: An Introduction to Curvature, GTM, Springer, $1^{\text {st }}$ Edition, 1997.
7. B. O' Neill, Semi-Riemannian Geometry with Applications to Relativity, Academic Press, New York, 1983.

## Course Title: Advanced Algebra

Course Code: MAT. 559

| $L$ | $T$ | $P$ | Credits |
| :---: | :---: | :---: | :---: |
| 3 | 0 | 0 | 3 |

Total Hours: 45

## Learning outcomes:

The students will be able to

- pursue research work in algebra.
- learn the concept of algebraic extensions of fields.
- explain the normal and separable extensions.
- apply the Galois theory to classical problems.

Algebraic extensions of fields: Irreducible polynomials and Eisenstein criterion, Adjunction of roots, Algebraic extensions, Algebraically closed fields

Activity: Students will use Eisenstein criteria to check the irreducibility of some polynomials.

## Unit II

## 11 Hours

Normal and separable extensions: Splitting fields, Normal extensions, Multiple roots, Finite fields, Separable extensions

Activity: Students will explore the applications of normal and separable extensions.

## Unit III

## 12 Hours

Galois theory: Automorphism groups and fixed fields, Fundamental theorem of Galois theory, Fundamental theorem of algebra.

Activity: Students will explore the applications of Galois theory..

## Unit IV

11 Hours
Applications of Galois theory to classical problems: Roots of unity and cyclotomic polynomials, Cyclic extensions, Polynomials solvable by radicals

Activity: Students will apply Galois theory to some classical problems.
TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. M. Artin, Algebra, 2 ${ }^{\text {nd }}$ Edition, Prentice Hall of India, Delhi, 2011.
2. P. B. Bhattacharya, S. K. Jain and S.R Nagpal, Basic Abstract Algebra, Cambridge University Press, New Delhi, 2003.
3. J. A. Gallian, Contemporary Abstract Algebra, Narosa Publishing House, New Delhi, 2008.
4. N. S. Gopalakrishnan, University Algebra, John Wiley \& Sons, 1986.
5. N. Herstein, Topics in Algebra, $2^{\text {nd }}$ Edition, Wiley Eastern Limited, New Delhi, 2006.
6. S. Surjeet and Q. Zameeruddin, Modern Algebra, $8^{\text {th }}$ Edition, Vikas Publishing House, New Delhi, 2006.
7. David Steven Dummit, and Richard M. Foote. Abstract algebra. Vol. 3. Hoboken: Wiley, 2004.

## Course Title: Discrete Differential Geometry

Course Code: MAT. 560


## Total Hours: 45

## Learning outcomes:

The students will be able to

- Understand combinatorial surfaces and their curvatures.
- Understand the exterior product and Differential forms.
- know about the curvature of Discrete surfaces and curvature flow.
- explain different designs and decompositions of surfaces.


## Unit-I

## 12 Hours

Combinatorial Surfaces: Abstract Simplicial Complex, Anatomy of a Simplicial Complex, Star, Closure, and Link, Simplicial Surfaces, Adjacency Matrices, Halfedge Mesh. The Geometry of Surfaces, Derivatives and Tangent Vectors, The Geometry of Curves, Curvature of Surfaces, Geometry in Coordinates.

Activity: Exercises on Simplicial complex, simplicial surfaces and curvature of surfaces.

## Unit-II

## 11 Hours

Exterior Algebra, Examples of Wedge and Star in $\mathbf{R}_{\mathrm{n}}$, Vectors and 1-Forms, Differential Forms and the Wedge Product, Hodge Duality, Differential Operators, Integration and Stokes' Theorem, Discrete Exterior Calculus.

Activity: Exercises on Wedge product, differential forms and Stoke's theorem.

## Unit-III

11 Hours

Curvature of Discrete Surfaces: Vector Area, Area Gradient, Volume Gradient, Other Definitions, Gauss-Bonnet, Numerical Tests and Convergence. The Laplacian: Basic Properties, Discretization via FEM, Discretization via DEC, Meshes and Matrices, The Poisson Equation, Implicit Mean Curvature Flow.

Activity: Exercises on Gradient, Divergence, Poisson equation and curvature flow.

## Unit-IV

## 11 Hours

Surface Parameterization: Conformal Structure, The Cauchy-Riemann Equation, Differential Forms on a Riemann Surface, Conformal Parameterization, Eigenvectors, Eigenvalues, and Optimization.

Activity: Exercises on CR-equations, Riemann surfaces and conformal parameterization.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. Alexander I. Boenko and Yuri B. Suris, Discrete Differential Geometry: Integrable Structure: 98 (Graduate Studies in Mathematics), American Mathematical Society; New ed. edition, 2009.
2. Alexander I. Bobenko, Advances in Discrete Differential Geometry Springer; Paperback, 2018.
3. Jiri Matousek, Lectures on Discrete Geometry: Springer- 212, GTM, 2nd edition, 2002.
4. Chuanming Zong, Strange Phenomena in Convex and Discrete Geometry, Springer (Universitext) Paperback 1st edition, 1996.

Course Title: Category Theory
Course Code: MAT. 564


Total Hours: 45

## Learning outcomes:

The students will be able to

- discuss the definition of Categories and Functors with examples.
- learn about Universal Mapping Properties and Free Objects
- learn about dual category, Epis, Monos, Equalizers, Co-Equalizers
- learn about Pullbacks and Categorical Limits
- learn about Natural Transformations and Adjunctions.


## Unit-I

Definition and examples of Categories: Sets, Pos, Rel, Mon, Groups, Top, Dis(X) Finite Categories. Isomorphic Objects. Constructions: Product of two categories, The Dual Category, The Arrow Category, The Slice and Co- Slice Category. The concept of functor and the category Cat. Free Monoids and their UMP. The UMP of Products. The Product Functor. The Free Monoid Functor.

Activity: Students will do examples/exercises of categories and Functors and explore the presence of other categories and UMPs.

## Unit-II

## 11 Hours

Contravariance and Duality, Functor of Several Variables, Covariant Representable Functor, Contravariant Representable Functor, Functors preserving Binary Products: The Canonical Comparison Arrow and the Necessary and Sufficient Condition for a Functor to preserve products. $\mathrm{H}^{\mathrm{A}}$ preserve products. Epis and mono, Initial and Terminal objects, Generalized elements.

Activity: Students will work on the exercises and examples on these topics.

## Unit-III

## 12 Hours

Coproducts. Coproduct of monoids. Equalizers, Equalizers as a monic, Generalized elements and equalizers, Coequalizers, Coequalizers as an epic. Coequalizer diagram for a monoid. Pullbacks, Properties of Pullbacks, Pullback as a functor. Limits, Cone to a diagram, limit for a diagram, Limits in terms of equalizers and pullbacks.

Activity: Students will do exercises/examples on these topics.

## Unit-IV <br> 12 Hours

Naturality, Examples of natural transformations. The Yoneda Lemma and its applications. Adjunction between categories, Hom-Set definition of adjoints. Unit and Co-Unit of Adjunction. Examples of Adjoints, Uniqueness up to isomorphism. Order Adjoints and interior operation in Topology as an order adjoint. Preservation of Limits (Co limits) by Right (Left) Adjoints.

Activity: Students will do examples of of Natural Transformations and Explore the occurrence of Adjunctions in Mathematics.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Recommended Books:

1. S. Awodey, Category Theory, (Oxford Logic Guides, 49, Oxford University Press.)
2. F. Borceux, Handbook of Categorical Algebra-1: Basic Category Theory, Cambridge University press, 1994.
3. S. Mac Lane, Categories for the working mathematicians, 2nd Edition, Springer 1971.
4. E. Riehl, Category theory in context, Aurora: Dover Modern math originals, 2017.

Course Title: Dynamical Systems
Course Code: MAT. 565

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credits |
| :---: | :---: | :---: | :---: |
| $\mathbf{3}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{3}$ |

## Total Hours: 45

## Learning outcomes:

The students will be able to

- know about dynamical Systems from an applied and practical point of view.
- how to compute the behavior of differential equations as parameters varies.
- know about the techniques that include bifurcation analysis and computation of normal forms, geometric methods, and the method of averaging.


## Unit-I

11 Hours
Basic Concepts: Discrete and continuous dynamical systems. Linear and nonlinear systems and principle of superposition. Linear and nonlinear forces. Concepts of evolution, iterations, orbits, fixed points, periodic and aperiodic (chaotic) orbits. Basics of Linear Algebra: Symmetric \& Skewsymmetric matrices, matrix norm and singular value decomposition. Eigenvalues, eigenvectors, and dynamical interpretation.

Activity: Students will apply some basics of linear algebra in study of dynamical systems.

## Unit-II

12 Hours

Canonical forms; simple and non-simple canonical systems. System of Equations.

## Stability Analysis:

Stability of a fixed point and classification equilibrium states (for both discrete and continuous systems). Concept of bifurcation and classification of bifurcations. Concepts of Lyapunov stability \& Asymptotic stability of orbits.

Activity: Students will check the stability of some dynamical systems by using the concept of bifurcation.

## Unit-III

## 11 Hours

Phase Portraits of various Linear and Nonlinear systems.
Hopf bifurcation. Concept of attractors and repellers, limit cycles and torus.

## Phenomena of Bifurcation:

Definition of bifurcation. Bifurcations in one, two and higher dimensional systems.

Activity: Students will draw the phase portraits of some dynamical systems using some software.

## Unit-IV

## 11 Hours

Hopf, Period doubling, Saddle node, Transcritical bifurcations. Feigenbaum's number. Local and Global bifurcations. Homoclinic $\&$ Hetero-clinic points and orbits. Poincaré-Bendixson Theorem. Conservative and Dissipative Systems.
Activity: Students will draw the phase portraits of some dynamical systems using some software and will study the orbits.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. P. G. Drazin, Nonlinear Systems, Cambridge University Press India.
2. R. L. Devaney, An Introduction to Chaotic Dynamical Systems, Addison Wesley, 1989.
3. Edward Ott, Chaos in Dynamical Systems, Cambridge University Press, 2002
4. G. L. Baker and J. P. Gollub, Chaotic Dynamics - An Introduction, Cambridge University Press, 1996.
5. J. Guckenheimer and P. Holmes, Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields, Springer, 1983.
6. L. Perko, Differential Equations and Dynamical Systems, Springer Verlag, 1991.
7. M. W. Hirsch and S. Smale, Differential Equations, Dynamical Systems and Linear Algebra, Academic Press, 174.
8. P. Hartman, Ordinary Differential Equations, 2nd edition, SIAM 2002.
9. C. Chicone, Ordinary Differential Equations with Applications, 2nd Edition, Springer, 2006.

## Course Title: Relativity Theory <br> Course Code: MAT. 566

| $L$ | $T$ | $P$ | Credits |
| :---: | :---: | :---: | :---: |
| 3 | 0 | 0 | 3 |

## Total Hours: 45

## Learning outcomes:

## Students will be able to

- Cover most modern topics of general relativity (GR) with some mathematical rigor.
- have solid working knowledge of GR,
- prepare themselves for research in relativistic astrophysics, cosmology and particle physics


## Unit-I

## 11 Hours

Introduction: The scope of the general theory of relativity, Geometry and physics, Space, time and gravity in Newtonian physics,

Spacetime and relativity : The Michelson-Morley interferometric experiment Postulates of special relativity, Lorentz transformations - The relativity of simultaneity - Length contraction and time dilation, Transformation of velocities and acceleration - Uniform acceleration - Doppler effect, Four vectors - Action for the relativistic free particle - Charges in an
electromagnetic field and the Lorentz force law, Conservation of relativistic energy and momentum

Activity: Students will explore the scope of general relativity.

## Unit-II

## 12 Hours

Tensor algebra and tensor calculus: Manifolds and coordinates - Curves and surfaces, Transformation of coordinates - Contravariant, covariant and mixed tensors - Elementary operations with tensors, The partial derivative of a tensor - Covariant differentiation and the affine connection, The metric Geodesics, Isometries - The Killing equation and conserved quantities, The Riemann tensor - The equation of geodesic deviation, The curvature and the Weyl tensors

Principles of general relativity: The equivalence principle - The principle of general covariance - The principle of minimal gravitational coupling.

Activity: Students will study how the equivalence principle works in real life.

## Unit-III

## 11 Hours

Field equations of general relativity: The vacuum Einstein equations, Derivation of vacuum Einstein equations from the action - The Bianchi identities, The stress-energy tensor - The cases of perfect fluid, scalar and electromagnetic fields, The structure of the Einstein equations

Activity: Students will obtain Einstein field equations in the form of partial differential equations.

## Unit-IV

## 11 Hours

Schwarzschild solution, and black holes: The Schwarzschild solution Properties of the metric - Symmetries and conserved quantities, Motion of particles in the Schwarzschild metric - Precession of the perihelion - Bending of light, Black holes - Event horizon, its properties and significance Singularities, The Kruskal extension - Penrose diagrams

Activity: Students will do further research about black holes.

## Suggested Textbooks:

1 L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields (Course of Theoretical Physics, Volume 2), Fourth Edition, Pergamon Press, New York, 1975.

2 B. F. Schutz, A First Course in General Relativity, Cambridge University Press, Cambridge, 1990.

3 R. d'Inverno, Introducing Einstein's Relativity, Oxford University Press, Oxford, 1992.

4 J. B. Hartle, Gravity: An Introduction to Einstein's General Relativity, Pearson Education, Delhi, 2003.

5 S. Carroll, Spacetime and Geometry, Addison Wesley, New York, 2004.
6 M. P. Hobson, G. P. Efstathiou and A. N. Lasenby, General Relativity: An Introduction for Physicists, Cambridge University Press, Cambridge, 2006.

7 S. Weinberg, Gravitation and Cosmology, John Wiley, New York, 1972.
8 J. V. Narlikar, An Introduction to Relativity, Cambridge University Press, 2010 (For the lectures on General Relativity and Cosmology).

Course Title: Review of Mathematical Concepts
Course Code: MAT. 567


Total Hours: 30

## Learning outcomes:

Students will be able to

- Review the basic concepts of Analysis.
- Understand the basic concepts of Advanced Analysis and Topology.
- Apply the techniques of Linear Algebra for solving problems.
- Review the concepts in Complex Analysis.


## Unit I

## 12 Hours

Analysis: Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, derivative as a
linear transformation, inverse and implicit function theorems. Metric spaces, compactness, connectedness. Normed linear Spaces. Spaces of continuous functions as examples.

Activity: Problem solving based on NET/GATE questions.
Linear Algebra: Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms

Topology: basis, dense sets, subspace and product topology, separation axioms, connectedness and compactness.

Activity: Problem solving based on NET/GATE questions.

## Unit II

## 11 Hours

Complex Analysis: Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

Activity: Problem solving based on NET/GATE questions.
Algebra: Permutations, combinations, pigeon-hole principle, inclusionexclusion principle, derangements. Fundamental theorem of arithmetic, divisibility in $Z$, congruences, Chinese Remainder Theorem, Euler's $\varnothing$ function, primitive roots. Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, Sylow theorems. Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions, Galois Theory.

## Unit III

## 11 Hours

Ordinary Differential Equations (ODEs): Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General theory of homogeneous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

Partial Differential Equations (PDEs): Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Activity: Problem solving based on NET/GATE questions.

## Unit IV

## 11 Hours

Numerical Analysis : Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and GaussSeidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

Calculus of Variations: Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations.

Linear Integral Equations: Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.

Activity: Problem solving based on NET/GATE questions.
TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

## Suggested Readings:

1. A. Kumar and S. Kumaresan, A Basic Course in Real Analysis, Narosa, Publishing House, 2014.
2. G. De Barra, Measure Theory and Integration, Ellis Horwood Limited, England, 2003.
3. H. L. Royden, Real Analysis, Macmillan, New York, 1988.
4. J. Gilbert and L. Gilbert, Linear Algebra and Matrix Theory, Cengage Learning, 2004.
5. J. R. Munkres, Topology- A First Course, Prentice Hall of India, New Delhi, 1975.
6. K. Hoffman and R. Kunze: Linear Algebra $2^{\text {nd }}$ Edition, Pearson Education (Asia) Pvt. Ltd/ Prentice Hall of India, 2004.
7. L. V. Ahlfors, Complex Analysis, Tata McGraw Hill, 1979.
8. M. A. Armstrong, Basic Topology, Springer, Paperback Edition, 2004.
9. P. B. Bhattacharya, S.K. Jain and S.R. Nagpaul, Basic Abstract Algebra, Wiley Eastern, Delhi, 2003.
10. S. Kumaresan, Topology of Metric Spaces, second edition, Narosa Publishing House New Delhi, 2015.
11. S. Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 2007.
12. W. Rudin, Principles of Mathematical Analysis, 3rd Edition, McGraw Hill, Kogakusha, International student Edition, 1976.
13. A. Pinckus, and S. Zafrany, Fourier series and Integral Transform, Cambridge University Press, New York, 1997.
14. G. D. Smith, Numerical Solution of Partial Differential Equations, Oxford: Clarendon Press, 1986.
15. I. N. Sneddon, Elements of Partial Differential Equations, McGraw-Hill, 2006.
16. J. A. Gallian, Contemporary Abstract Algebra, Narosa Publishing House, New Delhi, 2008.
17. L. C. Evans, Partial Differential Equations. Graduate Studies in Mathematics, American Mathematical Society, 2nd Edition, Indian Reprint, 2014.
18. M. D. Raisinghania,Advanced Differential Equations, S. Chand \& Company Ltd., New Delhi, 2001.
19. M. K. Jain, S.R.K. Iyengar and R.K. Jain, Numerical Methods for Scientific and Engineering Computation, 6th Edition, New Age International, New Delhi, 2015.
20. P. B. Bhattacharya, S.K. Jain and S.R Nagpal, Basic Abstract Algebra, Cambridge University Press, New Delhi, 2003.
21. R. L. Burden and J. D. Faires, Numerical Analysis, 9th Edition, Cengage Learning, 2011.
22. R. P. Kanwal, Linear Integral Equations, Birkhauser, Boston, 1996.
23. R. S. Gupta, Elements of Numerical Analysis, Cambridge University Press, 2nd Edition, 2015.
24. S. L. Ross, Differential Equations, Wiley, 1984.I. Miller and M. Miller, Mathematical Statistics, 6 ${ }^{\text {th }}$ Edition, Oxford \& IBH Pub., 1999.

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credits |
| :--- | :--- | :--- | :--- |
| $\mathbf{0}$ | $\mathbf{0}$ | 2 | $\mathbf{1}$ |

## Course Title: Basics of LateX (Practical)

## Code: MAT. 568

Total Hours: $\mathbf{3 0}$

## Laboratory outcomes:

The students will learn the:

1. Installation of the LaTeX software in Windows and Linux and understanding LaTeX compilation and LaTeX editors.
2. Basic syntax used in LaTeX.
3. Writing mathematical equations, Matrices, Tables, Inclusion of graphics into LaTeX file.
4. Page configurations: Title, Abstract, Keywords, Chapter, Sections and Subsections.
5. References and their citations.
6. Labeling of equations, Table of contents, List of figures.
7. Use of Packages: amsmath, amssymb, amsthm, amsfonts, graphic.
8. Use of document classes: Article, Report, Book, Beamer.
9. Applications of LaTeX in writing reports, books, research papers and thesis.

Transaction mode: Lecture/Demonstration/ Co Operative learning/ programming / Practical/ Group discussion/Team teaching/Experimentation/Tutorial/Problem solving/Self-learning.

## Suggested Readings:

1 D. F. Griffiths and D. J. Higham, Learning LaTeX, 2 nd Edition, Philadelphia, Pennsylvania, SIAM, 1997.
2 L. Lamport. LATEX: A Document Preparation System, User's Guide and Reference Manual. $2^{\text {nd }}$ Edition, Addison Wesley, New York, 1994.
3 M. Goossens, F. M. Michel, and S. Alexander, The LaTeX companion, $2^{\text {nd }}$ Edition, Addison-Wesley, 1994.

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credits |
| :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ |

## Course Title: Entrepreneurship

## Course Code: STA. 563

Total Hours: 15

## Learning Outcomes:

The students will be able to

- Understand the basic concepts of entrepreneur, entrepreneurship and its importance.
- Aware of the issues, challenges and opportunities in entrepreneurship.
- Develop capabilities of preparing proposals for starting small businesses.
- Know the availability of various institutional supports for making a new start-up.


## Unit - 1

## 3 Hours

Introduction to entrepreneur and entrepreneurship; Characteristics of an entrepreneur; Characteristics of entrepreneurship; entrepreneurial traits and skills; innovation and entrepreneurship; Types of entrepreneurial ventures; enterprise and society in Indian context; Importance of women entrepreneurship

## Unit - 2

5 Hours
Promotion of a venture - Why to start a small business; How to start a small business; opportunity analysis, external environmental analysis, legal requirements for establishing a new unit, raising of funds, and establishing the venture - Project report preparation - format for a preliminary project report, format for a detailed/final project report.

## Unit - 3

4 Hours
Mathematics and Statistics as a tool for innovation, current challenges to be tackled in marketing, health and environmental sectors, figuring out scientific needs of the society, mathematics and statistics in cross-disciplinary industries, examples of successful mathematics and statistics spin-offs, funding from scientific and governmental bodies.

## Unit - 4

3 Hours
Communicating with data and statistics, hypothesis testing and modeling, importance of units and measurement, from idea to research to product design and development, scope of innovation in: theoretical physics, String theory, mathematical modeling, optimization problems, Image Processing, Architecture, Machine Learning, Econometrics and many more.

## Suggested Readings:

1. Renu Arora, Entrepreneurship and Small Business, Dhanpat Rai \& Sons Publications, 2008.
2. Prasaaan Chandra, Project Preparation, Appraisal, Implementation, Tata Mc-Graw Hills, 2018.
3. Vasant Desai, Management of a Small Scale Industry, Himalaya Publishing House, 2019.
4. P. C. Jain, Handbook of New Entrepreneurs, Oxford University Press, 2015.
5. S. B. Srivastava, A Practical Guide to Industrial Entrepreneurs, Sultan Chand \& Sons, 2009.

## Course Title: Research Proposal

Course Code: MAT. 600

| $\mathbf{L}$ | $\mathbf{T}$ | $\mathbf{P}$ | Credits |
| :---: | :---: | :---: | :---: |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{8}$ | $\mathbf{4}$ |

Total Hours: 120

## Learning outcome:

The students will be able to

- develop interest in theoretical and practical research.
- decide their area of research as per their competency.
- get theoretical and practical knowledge of a specific area of research.
- prepare themselves for quality research in any mathematical discipline and allied areas.


## Evaluation Criteria:

| Dissertation Proposal |  |  |  |  |
| :--- | :--- | :--- | :--- | :---: |
|  | Marks | Proposal <br> $(30)$ | Presentation <br> $(10)$ | Viva-Voce <br> (10) |
| Supervisor | 50 |  |  |  |
| HoD and Senior | 50 |  |  |  |


|  |  |  |  |
| :--- | :--- | :--- | :--- |

Transaction mode: Lecture/Demonstration/ Co Operative learning/ programming / Practical/ Group discussion/Team teaching/Experimentation/Tutorial/Problem solving/Self-learning.

## M. Sc. Mathematics (Semester-IV)

Course Title: Dissertation
Course Code: MAT. 600

| $\mathbf{L}$ | T | $\mathbf{P}$ | Credits |
| :---: | :---: | :--- | :---: |
| $\mathbf{0}$ | $\mathbf{0}$ | 40 | 20 |

Total Hours: 600

## Learning outcome:

The students will be able to

- gain Theoretical and Practical knowledge of a specific area of research.
- have a good understanding of the subject to pursue research in reputed Institutions of higher learning.
- apply theoretical and practical knowledge to real life situations.
- prepare themselves for collaborative research in India and abroad.
- get Phd positions in reputed universities/institutes at regional/national/international level.


## Evaluation Criteria:

| Dissertation |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Continuous <br> Assessment | Report | Presentatio <br> n | Viva- <br> Voce | Total <br> Marks |  |
| Regularity <br> in work | Mid-term <br> evaluatio <br> n |  | 05 | 05 | 50 |  |
| Supervis <br> or | 10 | 10 | 20 | 10 | 10 | 50 |
| HoD, <br> Senior <br> faculty <br> member <br> and <br> external <br> expert | - | - | 30 |  |  |  |

Evaluation pattern similar to fourth semester dissertation will apply for internship where supervisor will award $50 \%$ marks and external co-supervisor, HoD and senior-most faculty will award 50\% marks.

Transaction mode: Lecture/Demonstration/ Co Operative learning/ programming / Practical/ Group discussion/Team teaching/Experimentation/Tutorial/Problem solving/Self-learning.

