Central University of Punjab, Bathinda

Course Scheme & Syllabus

for

Ph. D. Course Work

in

Mathematics

Department of Mathematics and Statistics
School of Basic and Applied Sciences

Course structure for Ph.D. Course work in Mathematics

Program Outcomes:

Learners will be able to:

- develop a broad understanding of recent theories, tools and techniques in research areas of Mathematics
- enable to be competent to apply various Mathematical techniques in variety of situations
- enable to independently develop and plan research in various areas of mathematics and allied areas.

Semester I

S. No.	Subject Code	Subject Name	Courses	Credit Hours			Credits
				L	T	P	
1.	STA.701	Research Methodology	Core	2	0	0	2
2	MAT.702	Computer Applications	Core	2	0	0	2
3.	MAT.703	Review Writing and Seminar	Core	0	0	0	2
4.	MAT.751	Research and Publication Ethics	Core	2	0	0	2
Opt any two out of the following elective courses offered							
5.	MAT.705	Symmetries and Differential Equations	DE	4	0	0	4
6.	MAT.706	Fractional Calculus	DE	4	0	0	4
7.	MAT.707	Advanced Partial Differential Equations	DE	4	0	0	4
8.	MAT.708	Differential Equations and Boundary-Value Problems	DE	4	0	0	4
9.	MAT.709	Differential Topology	DE	4	0	0	4
10.	MAT.710	Algebraic Topology	DE	4	0	0	4
11.	MAT.711	Riemannian Geometry	DE	4	0	0	4
12.	MAT.712	Riemann - Finsler Geometry	DE	4	0	0	4
13.	MAT.713	General Relativity	DE	4	0	0	4
		Total Credits					**16**

Course Title: Research Methodology
Course Code: MAT.701
Total Hours: 30

L	T	P	Cr
2	0	0	2

Learning outcomes:.
Upon successful completion of this course, the student will be able to:

1. Understand meaning, objectives, characteristics, significance, and types of research.
2. Understand the different steps of formulation of research problem.
3. Use latex to write different type of reports.
4. Understand the basics of different bibliography/reference preparation styles.

Unit-I **8 Hours**

Introduction: Meaning, Objectives, Characteristics, Significance, and Types of Research.

Formulating Research Problem: Understanding a Research Problem, Selecting the Research Problem, Steps in Formulation of a Research Problem, Formulation of Research Objectives, and Construction of Hypothesis.

Unit-II **8 Hours**

Installation of the software LaTeX, Understanding LaTeX compilation and LaTeX editors, Basic syntax, Writing mathematical equations, Matrices, Tables, Inclusion of graphics into LaTeX file.

Page configurations: Title, Abstract, Keywords, Chapter, Sections and Subsections, References and their citations, Labeling of equations, Table of contents, List of figures, List of tables, Page numbering, Generating index.

Unit-III **7 Hours**

Packages: amsmath, amssymb, amsthm, amsfonts, hyperrefer, graphic, color, xypic, latexsym, natbib, setspace, multicol, subcaption, url, verbatim, tikz, and geometry.

Classes: Article, Report, Book, Letter, Slides, Beamer.

Unit-IV **7 Hours**

Report Writing: Types of Reports – Technical and Popular Reports, Significance of Report Writing, Different Steps in Writing Report, Art of Writing Research Proposals, Research Papers, Project Reports, and Dissertations/Thesis; Basics of Citation and Bibliography/Reference Preparation Styles; Report Presentation: Oral and Poster Presentations of Research Reports.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/Seminar/Group discussion/Team teaching/Tutorial/Problem solving/ E-team teaching/Self-learning.

Suggested Readings:

1. Kothari, C.R. and G. Garg (2014): *Research Methodology*: Methods and Techniques, 3rd ed., New Age International Pvt. Ltd. Publisher
2. Kumar, R. (2014): *Research Methodology – A Step-By-Step Guide for Beginners*, 4th ed., Sage Publications
3. J. Anderson, *Thesis and Assignment Writing*, 4th ed., Wiley, USA, 2001.
4. Catherine Dawson, *Practical Research Methods,* New Delhi, UBS Publishers' Distributors, 2014.
5. L. Lamport. LATEX: A Document Preparation System, User's Guide and Reference Manual. 2nd Edition, Addison Wesley, New York, 1994.
6. Copyright Protection in India [website: http:copyright.gov.in].
7. World Trade Organization [website: www.wto.org].

Course Title: Computer Applications
Course Code: MAT.702

L	T	P	Cr
2	0	0	2

Total Hours: 30

Learning outcomes:

Upon successful completion of this course, the student will be able to:

1. Use different operating system and their tools easily.
2. Use word processing software, presentation software, spreadsheet software and latex.
3. Understand networking and internet concepts.
4. Use computers in every field like teaching, industry and research.

Course Contents

UNIT I **Hours: 8**

Computer Fundamentals: Introduction to Computer, Input devices, Output Devices, Memory (Primary and Secondary), Concept of Hardware and Software, C.P.U., System bus, Motherboard, Ports and Interfaces, Expansion Cards, Ribbon Cables, Memory Chips, Processors, Software: Types of Software, Operating System, User Interface of popular Operating System, Introduction to programming language, Types of Computer.

UNIT II **Hours: 7**

Computer Network: Introduction to Computer Network, Types of Network: LAN, WAN and MAN, Topologies of Network, Internet concept, WWW.

Word Processing: Text creation and Manipulation; Table handling; Spell check, Hyper-linking, Creating Table of Contents and table of figures, Creating and tracking comments, language setting and thesaurus, Header and Footer, Mail Merge, Different views, Creating equations, Page setting, Printing, Shortcut keys.

UNIT III **Hours: 8**

Presentation Tool: Creating Presentations, Presentation views, working on Slide Transition, Making Notes Pages and Handouts, Drawing and Working with Objects, Using Animations, Running and Controlling a Slide Show, Printing Presentations, and Shortcut keys.

Spread Sheet: Entering and editing data in cell, Basic formulas and functions, deleting or inserting cells, deleting or inserting rows and columns, printing of Spread Sheet, Shortcut keys.

UNIT IV **Hours: 7**

Use of Computers in Education and Research: Data analysis tools, e-Library, Search engines related to research, Research paper editing tools like Latex.

Transactional Modes:

PPT
Video
e-content
google drive

Suggested Readings:

1. Sinha, P.K. Computer Fundamentals. BPB Publications.
2. Goel, A., Ray, S. K. 2012. Computers: Basics and Applications. Pearson Education India.
3. Microsoft Office Professional 2013 Step by Step https://ptgmedia.pearsoncmg.com/images/9780735669413/samplepages/97807356694 13.pdf

Course Title: Review Writing and Presentation
Course Code: MAT.703
Total Hours: 60

L	T	P	Cr
0	0	4	2

Learning outcomes:.

Upon successful completion of this course, the student will be able to:

1. Understand the aspects of the Review writing and seminar presentation.
2. Write a review of existing scientific literature with simultaneous identification of knowledge gaps.
3. Identify the predatory publications and open access publications.

Evaluation Criteria:

The evaluation criteria for "Review Writing and Presentation" shall be as follows:

S. No.	Criteria	Marks
1.	Literature review report	20
2.	Content of presentation	10
3.	Presentation Skills	10
4.	Handling of queries	10
Total		**50**

Course Title: Research and Publication Ethics
Course Code: MAT.751
Total Lectures: 30

L	T	P	Cr
2	0	0	2

Learning outcomes:.
Upon successful completion of this course, the student will be able to:
1. Understand the philosophy and value of publication ethics.
2. Understand ethics with respect to science and research.
3. Identify the predatory publications and open access publications.
4. Use different software and their tools to check plagiarism check.

Unit-I **8 hours**

PHILOSOPHY AND ETHICS: Introduction to philosophy definition, nature and scope, concept, branches ; Ethics definition, moral of moral judgements and reactions.

SCIENTIFIC CONDUCT: Ethics with respect to science and research; Intellectual honesty and research integrity; Scientific misconducts Falsification, Fabrication, and plagiarism (FFP); Redundant publication duplicate and overlapping publication, salami slicing; Selective reporting and misrepresentation of data

Unit-II **7 Hours**

PUBLICATION ETHICS: Publication ethics definition introduction and importance; Best practices / standards setting initiatives and guidelines: COPE, WAME, etc.; Conflicts of interest; Publication misconduct: definition, concept, problems that lead to unethical behaviour and vice versa, types; violation of publication ethics , authorship and contributor ship; identification of publication misconduct, complaints and appeals; predatory publishers and journals

Unit-III **8 hours**

OPEN ACCESS PUBLISHING: Open access publication and initiatives; SHERPA/Ro MEO online resource to check publisher copyright & self-archiving policies; software tool to identify predatory publications developed by SPPU; journal finder/ journal suggestion tools viz. JANE, Elsevier Journal Finder, Springer Journal Suggester, etc.

PUBLICATION MISCONDUCT:

A. Group Discussions : a) Subject specific ethical issues, FFP, authorship; b) Conflicts of interest; c) Complaints and appeals: examples and fraud from India and abroad

B. Software tools: Use of plagiarism software like Turnitin, Urkund and other open source software tools.

Unit-IV **7 Hours**

DATABASES AND RESEARCH METRICS:

A. Databases : a) Indexing databases; b) Citation databases: web of Science, Scopus, etc.

B. Research Metrics: a) Impact Factor of journal as per journal Citation Report, SNIP, SJR, IPP, Cite Score b) Metrics, h-index, g-index, i10 index, altmetrics

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning/Practical.

Suggested Readings:

1. Melville, S., and Goddad, W. (1996). Research Methodology: An Introduction to Science and Engineering students. South Africa: Juta Academic.
2. Kothari, C.R. and G. Garg (2014): *Research Methodology*: Methods and Techniques, 3rd ed., New Age International Pvt. Ltd. Publisher.

Course Title: Symmetries and Differential Equations
Course Code: MAT.705
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning outcomes:.
Upon successful completion of this course, the student will be able to:

1. know the basics of dimensional analysis and Lie group of transformations.
2. Learn the basic applications of Lie group of transformation for solving ordinary and partial differential equations.
3. Learn the concept of extended infinitesimal transformations.
4. Understand the basic concept of multi-parameter Lie groups of transformations.

Unit I **15 Hours**

Dimensional Analysis: Buckingham Pi-theorem, Assumptions behind dimensional analysis, Conclusions from dimensional analysis, Proof of the Buckingham Pi-theorem and examples, Application of dimensional analysis to partial differential equations, Generalization of dimensional analysis, Invariance of partial differential equations under scaling of variables

Unit II **15 Hours**

Lie Group of Transformations: Groups, Examples of groups, Groups of transformations, One-parameter Lie group of transformations, Examples of one-parameter Lie groups of transformations, Infinitesimal transformations: First fundamental theorem of Lie, Infinitesimal generators, Invariant functions

Unit III **15 Hours**

Canonical coordinates, Invariant surfaces, Invariant curves, Invariant points, Extended transformations: Extended group transformations-one dependent and one independent variable, Extended infinitesimal transformations-one dependent and one independent variable, Extended transformations-one dependent and n independent variables

Unit IV **15 Hours**

Multi-parameter Lie groups of transformations; Lie algebras, r-parameter Lie groups of transformations, Lie algebras, Examples of Lie algebras, Solvable Lie

algebras

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/ Problem solving/ E-team teaching/Self-learning.

Suggested Readings:

1. G. W. Bluman and A. C. Anco, *Symmetry and Integration Methods for Differential Equations*, Appl. Math. Sci., 154, Springer, New York, 2002.
2. G. W. Bluman and S. Kumei, *Symmetries and Differential Equations*, Appl. Math.Sci., Springer-Berlin, 1989.
3. P. J. Olver, *Applications of Lie Groups to Differential Equations*, Springer-Verlag, New York 1993.
4. L. V. Ovsiannikov, *Group Properties of Differential Equations*, Novosibirsk, Moscow, 1962.

Course Title: Fractional Calculus
Course Code: MAT.706
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning outcomes:.
Upon successful completion of this course, the student will be able to:
1. learn the basics of fractional calculus.
2. know the basics of fractional differential equations.
3. Learn the different types of fractional derivatives
3. Understand the concept of linear fractional differential equations.
4. apply different techniques for solving fractional differential equations.

Unit I **16 Hours**

Special Functions of Fractional Calculus: Gamma function, Some properties of Gamma function, Beta function, Contour integral representation. Fractional derivatives and integrals, GrunwaldLetnikov Fractional derivatives, Riemann-Liouville fractional derivatives, Caputo's fractional derivative, The Leibniz rule for fractional derivatives, Geometric and physical interpretation of fractional integration and fractional differentiation.

Unit II **14 Hours**

Sequential fractional derivatives. Left and right fractional derivatives. Properties of fractional derivatives. Laplace transforms of fractional derivatives. Fourier transforms of fractional derivatives. Mellin t
ransforms of fractional derivatives.

Unit III **15 Hours**

Linear Fractional Differential Equations: Fractional differential equation of a general form. Existence and uniqueness theorem as a method of solution. Dependence of a solution on initial conditions. The Laplace transform method. Standard fractional differential equations. Sequential fractional differential equations.

Unit IV **15 Hours**

Fractional Differential Equations: Introduction, Linearly independent solutions, Solutions of the homogeneous equations, Solution of the non-homogeneous fractional differential equations, Reduction of fractional differential equations to ordinary differential equations. Semi differential equations

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:

1. K. B. Oldham & J. Spanier, *The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order*, Dover Publications Inc, 2006.
2. K. S. Miller & B. Ross., *An Introduction to the Fractional Calculus and Fractional Differential Equations Hardcover*, Wiley Blackwell, 1993.
3. I. Podlubny, *Fractional Differential Equations*, Academic Press, 1998

Course Title: Advanced Partial Differential Equations

Course Code: MAT.707

Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning outcomes:.

The students will be able to

- Review the basic concepts of Distribution and Sobolev Spaces
- Explain the Weak Solutions of Elliptic Boundary Value Problems.
- Explain the concept of Fourier transform and Evolution Equations.
- Study of the Calculus of Variations for Euler-Lagrange equation

Unit-I **16 Hours**

Distribution: Test functions and distributions, examples, operations on distributions, supports and singular supports, convolution, fundamental solutions, fourier transform, Schwartz space, tempered Distributions.

Sobolev Spaces: Basic properties, approximation by smooth functions, extension theorems, compactness theorems, dual spaces, functional order spaces, trace spaces, trace theory, inclusion theorem.

Unit-II **15 Hours**

Weak Solutions of Elliptic Boundary Value Problems: variational problems, weak formulation of elliptic PDE, regularity, Galerkin method, Maximum principles, eigenvalue problems, introduction to finite element methods.

Unit-III **14 Hours**

Evolution Equations: unbounded linear operators, C_0 – semigroups, Hille-Yosida theorem, contraction semigroup on Hilbert spaces, heat equation, wave equation, Schrodinger equation, inhomogeneous equations.

Unit-IV **15 Hours**

Calculus of Variations: Euler-Lagrange equation, second variation, existence of minimizers (coactivity, cower semi-continuity, convexity), regularity, constraints (nonlinear eigenvalue problems, variational inequalities, harmonic maps, incompressibility), critical points (mountain pass theorem and applications to elliptic PDE).

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Books:

1. S. Kesavan, *Topics in Functional Analysis and Application*, Wiley-Eastern, New International, New Delhi, 1999.
2. L. C. Evans, *Partial Differential Equations. Graduate Studies in Mathematics*, American Mathematical Society, 2nd Edition, Indian Reprint, 2014.
3. Rao, K. S., *Introduction to Partial Differential Equation*, 2nd Edition, PHI Learning Pvt. Ltd. 2010.
4. Amarnath, T., *An Elementary Course in Partial Differential Equations*, 2nd Edition, Narosa Publishing House 2012.
5. Sneddon, I. N., *Elements of Partial Differential Equations*, McGraw-Hill Book Company, New York 1988.

Course Title: Differential Equations and Boundary-Value Problems

Course Code: MAT.708

Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning outcomes:.

The students will be able to

- Review the basic concepts of Existence and uniqueness of solutions of ODEs
- Explain the stability and Liapunov functions for Non-linear system of ODE.
- Classify the First-order PDEs, into cauchy problem and second order PDEs into Parabolic, Hyperbolic and Elliptic.
- Review the basic concepts of Sobolev Spaces and system of conservation laws.

Learning Outcomes: The objective of this course is to ensure that a student learns basics differential equations and boundary value problems.

Unit I **16 Hours**

Existence and uniqueness of solutions of ODEs, power series solution, singular points, some special functions. Non-linear system of ODE: preliminary concepts and definitions, the fundamental existence-uniqueness results, dependence on initial conditions and parameters, the maximum interval of existence.

Unit II **15 Hours**

Linearlization, stability and Liapunov functions, saddle, nodes, foci and centers,

normal form theory and Hamiltonian systems. Boundary value problems: Green's function method, Sturm-Liouville problem.

Unit III **14 Hours**

First-order PDEs, cauchy problem, method of characteristics, second-order PDEs, classification, characteristics and canonical forms. Elliptic boundary value problems: maximum principle, Green's function,

Unit IV **15 Hours**

Sobolev spaces, variational formulations, weak solutions, Lax-Milgram theorem, trace theorem, Poincarénequality, energy estimates, Fredholm alternative, regularity estimates, system of conservation laws, entropy criteria.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:

1. L. Perko, *Differential Equations and Dynamical Systems*, 3rd Edition Springer-Verlag New York, 2001.
2. J. Guckenheimer, P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, 1st Edition, Springer-Verlag, New York, 1983.
3. S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, 2nd Edition, Springer-Verlag, New York, 1990.
4. L. C. Evans, *Partial Differential Equations, Graduate Studies in Mathematics*, Vol. 19, American Mathematical Society, Providence, 1998.
5. R. C. McOwen, *Partial Differential Equations-Methods and Applications*, 2nd Edition Pearson Education Inc., Indian Reprint 2002.
6. S. J. Farlow, *Partial Differential Equations for Scientists and Engineers*, Dover Publications, New York, 1982.

Course Title: Differential Topology
Course Code: MAT.709
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes:

After the completion of the course students will be able to

1. learn basics of Differential Topology.
2. appreciate better the topics covered in allied courses like Algebraic Topology, Riemannian geometry and Riemann-Finsler geometry
3. adequately prepared for pursuing research in above mentioned topics

UNIT-I **15 Hours**

Topological manifolds, Charts, Atlases, Smooth manifolds, Examples of smooth manifolds, Manifolds with boundary, Smooth functions on a manifold, Smooth

maps between manifolds, Diffeomorphism, Smoothness in terms of components, Examples of smooth maps, Partial derivatives, and the Inverse function theorem.

UNIT-II **15 Hours**

Tangent space and tangent bundle, The Differential of a map, Chain rule, Bases for the tangent space at a point, Curves in a manifold, Submersions, Immersions and embeddings, Smooth covering maps, Critical and regular points, Submanifolds, Rank of a smooth map, Submersion and immersion theorems, Bump functions and partition of unity, Sard's theorem, The Whitney embedding theorem (statement only).

UNIT-III **15 Hours**

Vector fields and Lie bracket. Topological groups, Lie groups: Definition and examples, The product of two Lie groups, Lie subgroups, One parameter subgroups and exponential map, Homomorphism and isomorphism in Lie groups, Lie transformation groups, The tangent space and left invariant vector fields of a Lie group.

UNIT-IV **15 Hours**

Tensor algebra, Differential forms, Cotangent spaces, Pullback of l-forms, k-forms, Exterior product, Differential forms on a circle, Exterior derivative, Exterior algebra and Lie derivative, Global formulas for the Lie and exterior derivatives.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:

1. W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, 2nd edition, Academic Press, New York, 2003.
2. S. S. Chern, W. H. Chen and K. S. Lam, *Lectures on Differential Geometry*, World Scientific Publishing Co. Pvt. Ltd., 2000.
3. L. Conlon, *Differentiable Manifolds*, 2nd edition, Birkhauser Boston, Cambridge, MA, 2001.
4. N. J. Hicks, *Notes of Differential Geometry*, D. Van Nostrand Reinhold Company, New York, 1965.
5. S. Kumaresan, *A Course in Differential Geometry and Lie Groups (Texts and Readings in Mathematics),* Hindustan Book Agency, 2002.
6. J. M. Lee, *Introduction to Smooth Manifolds*, GTM, Vol. 218, Springer, New York, 2003.
7. L. W. Tu, *An Introduction to Manifolds*, Second edition, Springer, 2011.
8. F. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, Springer, New York, 1983.

Course Title: Algebraic Topology
Course Code: MAT.710
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning outcomes:
The students will be able to

- Understand the fundamental concept of Homotopy theory to pursue research.
- Develop the knowledge of fundamental group and covering spaces with related properties.
- Construct homotopies and prove homotopy equivalence.
- apply his or her knowledge of algebraic topology to formulate and solve problems of a geometrical and topological nature in mathematics.

Unit-I **14 Hours**

The Fundamental group: Homotopy of paths, Homotopy classes, The Fundamental group, Change of base point, Topological invariance, Covering spaces, The Fundamental group of the circle.

Unit-II **14 Hours**

Retractions and fixed points, No Retraction Theorem, The Fundamental theorem of Algebra, The Borsuk-Ulam theorem, The Bisection theorem, Deformation Retracts and Homotopy type, Homotopy invariance.

Unit-III **16 Hours**

Direct sums of Abelian Groups, Free products of groups, Uniqueness of free products, Least normal subgroup, Free groups, Generators and relations, The Seifert-Van Kampen theorem, The Fundamental group of a wedge of circles.

Unit-IV **16 Hours**

Classification of covering spaces: Equivalence of covering spaces, The general lifting lemma, The universal covering space, Covering transformation, Existence of covering spaces.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:

1. M. A. Armstrong, *Basic Topology,* UTM Springer, 2000.
2. S. Deo, *Algebraic Topology: A Primer (Texts and Readings in Mathematics),* Hindustan Book Agency, 2003.
3. M. J. Greenberg and J. R. Harper, *Algebraic Topology:* A First Course, 2nd Edition, Addison-Wesley Publishing Co, 1997.
4. A. Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.
5. W. S. Massey, *A Basic Course in Algebraic Topology,* SPRINGER (SIE), 2007.
6. J. R. Munkres, *Elements of Algebraic Topology*, Perseus Books, 1995.

7. J. J. Rotman, *An Introduction to Algebraic Topology*, *Text in Mathematics, No. 119*, Springer, New York, 2004.
8. E. H. Spanier, *Algebraic Topology (2nd edition*), Springer-Verlag, New York, 2000.

Course Title: Riemannian Geometry
Course Code: MAT.711
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning outcomes:
After the completion of the course students will be able to
1. learn basic concepts of Riemannian geometry.
2. appreciate better the topics covered in allied courses like, Riemann-Finsler geometry, Mathematical Physics and their applications in allied areas
3. adequately prepared for pursuing research in above mentioned topics.

Unit-I **16 Hours**
Review of differentiable manifolds and vector fields, Covariant differentiation of vector fields and affine connection, Riemannian metric, Riemannian manifolds, Riemannian connection, Fundamental theorem of Riemannian geometry via Koszul's formula.

Unit-II **14 Hours**
Tensors and tensor fields (Riemannian metric as the most significant example), Tensorial property, Covariant differentiation of tensor fields, Riemann curvature tensor, Ricci tensor, Sectional, Ricci and scalar curvatures, Isometries, Notion of covering spaces, Pull-back metrics via diffeomorphisms.

Unit-III **16 Hours**
Covariant differentiation of a vector field along a curve with specific examples, Arc length and energy of a piecewise smooth curve, Geodesics as length minimizing curves, First variation of arc length, To show that geodesics are critical points of the fixed end point first variation formula, Exponential map, Geodesic completeness, Geodesic normal coordinates, Hopf-Rinow theorem (statement only), Geodesic variations, Jacobi fields and Gauss lemma.

Unit-IV **14 Hours**
Second variation formula, The index form (Jacobi fields as minimizers of the index form), Global differential geometry, Spaces of constant sectional curvature, Bonnet-Myers theorem, Cartan-Hadamard theorem, Cartan's theorems (on determination of metric by curvature).

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:
1. M. Berger, *A Panoramic View of Riemannian Geometry*, Springer; 1st Edition, 2003. Corr. 2nd printing, 2007.

2. W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, 2nd Edition, Academic Press, New York, 2003.
3. S. S. Chern, W. H. Chen and K. S. Lam, *Lectures on Differential Geometry*, World Scientific Publishing, 2000.
4. M. P. Docarmo, *Riemannian Geometry*, Birkhausker Boston, 1992.
5. S. Kumaresan, *A Course in Differential Geometry and Lie Groups (Texts and Readings in Mathematics),* Hindustan Book Agency, 2002.
6. J. M. Lee, *Riemannian Manifolds: An Introduction to Curvature*, GTM, Springer, 1st Edition, 1997.
7. B. O' Neill, *Semi-Riemannian Geometry with Applications to Relativity*, Academic Press, New York, 1983.

Course Title: Riemann-Finsler Geometry
Course Code: MAT.712
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes: The objective of this course is to enable the students, concepts of Riemann-Finsler geometry so that they can pursue research in this area.

Learning outcomes:
After the completion of the course students will be able to
1. learn basic concepts of, concepts of Riemann-Finsler geometry.
2. appreciate better the topics covered in allied courses like, Riemannian geometry, Mathematical Physics and their applications in allied areas.
3. adequately prepared for pursuing research in Riemann-Finsler geometry.

UNIT I **15 Hours**

Minkowski norms, Euler's theorem, Fundamental inequality and its interpretation, Finsler structures: definitions and conventions, Examples: Minkowski and locally Moinkowski spaces, Riemannian manifolds, Randers spaces, Berwald spaces, Finsler spaces of constant flag curvature. Fundamental metric tensor and Cartan tensor.

UNIT II **15 Hours**

Vector bundle, Nonlinear connection on slit tangent bundle, Chern connection, Structure equations, Horizontal and vertical covariant derivatives, hh-, hv-, vv-curvatures, Bianchi identities and their consequences, Ricci identities, Geodesic spray coefficients, Flag curvature: Definition, example and its predecessor, Schur's lemma.

UNIT III **15 Hours**

Rund's differential equation and its consequence, Criterion for strong convexity, Berwald frame, Moore frame, Geometrical setup on sphere bundle, Cartan scalar(I), Landsberg scalar(J) and Gaussian curvature(K), Riemannian arc length of indicatrix, Gauss Bonnet theorem for Landsberg surfaces.

UNIT IV **15 Hours**

Geodesics: Sprays, shortest paths. Projectively equivalent Finsler metrics, Projectively flat metrics, Parallel vector fields, Parallel translations, Berwald metrics, Landsberg metrics, Distortion and S-curvature, Randers metric of isotropic S-curvature.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/Problem solving/E-team teaching/Self-learning.

Suggested Readings:

1. P. L. Antonelli (ed.), *Handbook of Finsler Geometry*, Kluwer Academic Publishers, Dordrecht, The Netherlands, 2003.
2. D. Bao, S. S. Chern, Z. Shen, *An Introduction to Riemann Finsler Geometry, Graduate texts in Mathematics 200*, Springer-Verlag, New York, 2000.
3. Xinyue Cheng and Zhongmin Shen, *Finsler geometry-An Approach via Randers spaces*, First Edition, Springer Berlin Heidelberg, 2012.
4. S. S. Chern and Z. Shen, *Riemann Finsler Geometry*, Nankai Tracts in Mathematics, Vol. 6. World Scientific Publishing Co. Pvt. Ltd., 2005.
5. M. Matsumoto, *Foundations of Finsler Geometry and Special Finsler Spaces*, Kaisheisha press, Saikawa, Otsu, 520, Japan, (1986).
6. Z. Shen, *Lectures on Finsler geometry*, World Scientific Press, 2001.
7. Y. B. Shen and Z. Shen, *Introduction to Modern Finsler Geometry*, World Scientific Publishing Co. Pvt. Ltd., 2016.

Course Title: General Relativity
Course Code: MAT.713
Total Hours: 60

L	T	P	Cr
4	0	0	4

Learning Outcomes:

The aim of this course is to make the students learn basic concepts of General theory of Relativity, so as to enable the students to pursue research work in this area, Einstein field equations, Cosmology and allied areas.

Unit-I

Transformation of coordinates, Tensor Algebra, Smooth manifolds: Definition and examples, vector fields, Lie brackets and Lie derivatives. Riemannian metric, parallel transport, covariant derivative, affine connection, Riemannian connection, Geodesics, Riemann curvature tensor and its symmetric properties, Ricci tensor, Bianchi identities, Einstein tensor.

Unit-II

Postulates of Special Theory of Relativity, Lorentz Transformation and consequences, Minkowski Diagram, Four dimensional spacetime continuum, Four vector formulation. Transition of Special to General Theory of Relativity, Principle of covariance, Equivalence Principle and consequences, Stress Energy Tensor, Einstein Field Equations and Newtonian limit.

Unit-III

Spherically symmetric solution to Einstein Field Equation in free space and in matter, Schwarzshild line element. Schwarzschild Black Holes. Equation of Planetary Orbits, Crucial tests of General Theory of Relativity, Advance of Perihelion, Gravitational bending of light and Gravitational Redshift.

Unit-IV

Cosmology: Large scale structure of Universe, Galactic Densities and the darkness of the Night Sky, Galactic Number Counts, Olber's paradox, Cosmological principles, Relativistic Universe and models. Einstein and de-Sitter models of static universe, Dynamical Universe, Comoving time, Red Shifts and Horizons, Friedmann-Robertson-Walker line element, Open and Closed Universe, Hubbles law, Early Universe.

TRANSACTION MODE: Lecture/Demonstration/Project Method/ Co Operative learning/ Seminar/Group discussion/Team teaching /Tutorial/ Problem solving/ E-team teaching/Self-learning.

Suggested Readings

1. R. Adler, M. Bazin and M. Schiffer, Introduction to General Relativity, McGraw Hill, 1965.
2. S. Caroll, Spacetime and geometry: an introduction to general relativity, Addison Wesley, 2004.
3. J. B. Hartle, Gravity: an introduction to Einstein's general relativity, Pearson education, 2003.
4. R. D. Inverno, Introducing Einstein's relativity, Oxford university press, 2005.
5. C. W. Misner, K. S. Thorne and J. A. Wheeler, Gravitation, W. H. Freeman and Co. 1973.
6. J. V. Narlikar, General Relativity and Cosmology, Macmillan, 1978.
7. A. S. Ramsey, Newtonian Attraction, Cambridge University Press, 1964.
8. B. F. Schutz, A First Course in General Relativity, Cambridge University Press, 2012.
9. S. Weinberg, Gravitation and cosmology: principles and applications of the general theory of relativity, John wiley and Sons, 2004.