Central University of Punjab

Bathinda

Course Scheme & Syllabus
for
Course Work
of
Ph.D in Mathematics / Statistics

1st SEMESTER

Syllabi Applicable for Admissions in 2015

Course structure for Ph.D. in Mathematics / Statistics

Students can move into the Ph.D programme after successful completion of one semester course work, provided they meet the requirements specified by the university.

Structure for course work for PhD in Mathematics / Statistics

	Semester I							
	S.No.	Subject Code	Subject Name	Credit Hours			Maximum Marks
				Theory	Practical	Total	
Compulsory Courses	1.	MAT.701	Research Methodology and Statistics	4		4	100
	2.	MAT.702	Review Writing and Presentation		8	4	100
	3.	MAT.703	Computer applications	4		4	100
	Opt any two out of the following elective courses offered						
Elective Courses	5.	MAT.704	Analysis	4		4	100
	6.	MAT.705	Symmetries and Differential Equations	4		4	100
	7.	MAT.706	Fractional Calculus	4		4	100
	8.	MAT.707	Advance theory of Partial Differential Equations	4		4	100
	9.	MAT.708	Advanced Numerical Analysis	4		4	100
	10.	MAT.709	Operations Research	4		4	100
	11.	MAT.710	Linear Algebra	4		4	100
	12.	MAT.711	Probability Theory	4		4	100
	13.	MAT.712	Number Theory	4		4	100
	14.	MAT.713	Advanced Algebra	4		4	100
	15.	MAT.714	Functional Analysis	4		4	100
	16.	MAT.715	Complex Analysis	4		4	100
	17.	MAT.716	Differential Equations and Boundary-Value Problems	4		4	100
	18.	MAT.717	Potential Flow of Fluids and Water-wave Theory	4		4	100
	19.	MAT.718	Stochastic Processes and Queueing Theory	4		4	100
	20.	MAT.719	Reliability Theory	4		4	100
	21.	MAT.720	Sampling Theory	4		4	100
	Total					20	500

Semester I

Course Title: Research Methodology and Statistics
Paper Code: MAT.701
Total Lectures: 56

L	T	P	Credits	Marks
4	0	0	4	100

Course Objective: The objective of this subject is to ensure that a student learns basis of scientific research and statistical methods to arrive at and verify the conclusions drawn.

Unit I **(10 Lecture Hours)**
General principles of research: Meaning and importance of research, Critical thinking, Formulating hypothesis and development of research plan, Review of literature, Interpretation of results and discussion. Technical writing: Scientific writing, Writing synopsis, Research paper, Poster preparation and Presentations and Dissertation.

Unit II **(15 Lecture Hours)**
General Statistics: Difference between parametric and non-parametric statistics, Univariant and multivariant analysis, Confidence interval, Errors, Levels of significance, Hypothesis testing. Measures of central tendency and dispersal, Histograms, Probability distributions (Binomial, Poisson and Normal), Sampling distribution, Kurtosis and skewness

Unit III **(16 Lecture Hours)**
Comparative Statistics: Comparing means of two or more groups: Student's t-test, Paired t-test, Mann-Whitney U-test, Wilcoxon signed-rank, One-way and two-way analysis of variance (ANOVA), Critical difference (CD), Fisher's LSD (Least significant difference), Kruskal–Wallis one-way ANOVA by ranks, Friedman two-way ANOVA by ranks, Chi-square test

Unit IV **(15 Lecture Hours)**
Regression and correlation: Standard errors of regression coefficients, Comparing two regression lines, Pearson Product - Moment Correlation Coefficient, Spearman Rank correlation coefficient, Power and sampling size in correlation and regression.

Recommended Books:
1. Gupta, S. (2008). Research methodology and statistical techniques. Deep & Deep Publications (P) Limited, New Delhi.
2. Kothari, C. R. (2014). Research methodology (s). New Age International (p) Limited. New Delhi.
3. Sahay, Vinaya and Pradumna Singh (2009). Encyclopedia of Research Methodology in life sciences. Anmol Publications. New delhi
4. Kauda J. (2012). Research Methodology: A Project Guide for University Students. Samfunds literature Publications.
5. Dharmapalan B. (2012). Scientific Research Methodology. Narosa Publishing House ISBN: 978-81-8487-180-7.
6. Norman, G. and Streiner, D. (2008). Biostatistics: The Bare Essentials.3/e (with SPSS). Decker Inc. USA.
7. Rao, P. P., S. Sundar and Richard, J. (2009). Introduction to Biostatistics and Research
8. Methods. PHI learning. 11. Christensen, L. (2007). Experimental Methodology. Boston: Allyn & Bacon.

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Course Title: Review Writing and Presentation
Paper Code: MAT.702

L	T	P	Credits	Marks
0	0	8	4	100

Objective: The objective of this course would be to ensure that the student learns the aspects of the Review writing and seminar presentation. Herein the student shall have to write a review of existing scientific literature with simultaneous identification of knowledge gaps that can be addressed through future work.

The evaluation criteria for "Review Writing and Presentation" shall be as follows:

Maximum Marks: 100

S.No.	**Criteria**	**Marks**
1.	Review of literature	25
2.	Identification of gaps in knowledge	15
3.	References	10
4.	Content of presentation	15
5.	Presentation Skills	20
6.	Handling of queries	15
Total		100

Course Title: Computer Applications
Paper Code: MAT.703
Total Lectures: 56

L	T	P	Credits	Marks
4	0	0	4	100

Objectives: The objective of this course is to develop understanding of different software and hardware systems available in industry among the participants and to build up the experience of computer usage in business organizations with specific reference to commercial data processing systems.

Unit I **(14 Lecture Hours)**

Fundamentals of Computers: Block Diagram of Computer, Hardware Components, Introduction to computer network and World Wide Web, Sharing Data over Network, Internet Terminology, Searching over Internet, Google: advance Search Operations, Email, Checking Plagiarism using Internet

Unit II **(14 Lecture Hours)**

Introduction to Word Processing and Microsoft Office, Creating and Saving Documents, Text Formatting, Tables, Document Review Option, Mail Merge, Inserting Table of Contents, Reference Management.

Introduction to Spreadsheet and Microsoft Excel, Text Formatting, Formulas, Charts, Table formatting, Sorting Records, Filtering the content.

Unit III **(14 Lecture Hours)**

Computer Configuration, Memory Hierarchy, Software Structure, Introduction to Operating System, Operating System types and functions. Introduction to Disk Operating System, DOS Internal and External Commands, Introduction to Windows operating System, Windows Task Manger.

Unit IV **(14 Lecture Hours)**

Introduction to MS Paint, Figure Designing components in MS Paint

Introduction to Microsoft PowerPoint, Layout Selection, Designing and Formatting Slides, Slide Design and background formatting, Bullets and Numbering, Transition Style, Custom Animations, Hyperlink to Local files and Web Pages, Movies and Sound, Slide Timings.

Recommended Books:

1. Gookin, D. (2007). MS Word for Dummies. Wiley.

2. Harvey, G. (2007). MS Excel for Dummies. Wiley

3. Sinha, P.K., Computer Fundamentals, BPB Publications.

Course Title: Analysis
Paper Code: MAT.704
Total Lectures: 56

L	T	P	Credits	Marks
4	0	0	4	100

Unit I **(15 Lecture Hours)**

Elementary set theory: Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, uniform convergence, Continuity, uniform continuity, differentiability, mean value theorem. Metric spaces: definition and examples, open and closed sets, Compact sets, elementary properties of compact sets, k- cells, compactness of k-cells, compact subsets of Euclidean space R^{k}, Perfect sets, Cantor set, Separated sets, connected sets in a metric space, connected subsets of real line.

Unit II **(14 Lecture Hours)**

Convergent sequences (in Metric spaces), Cauchy sequences, subsequences, Complete metric space, Examples of complete metric space, Limits of functions (in Metric spaces), Continuous functions, continuity and compactness, Continuity and connectedness. Separable Metric spaces. Cantor's intersection theorem, category of a set and Baire's category theorem, Banach contraction principle.

Unit III **(13 Lecture Hours)**

Functions of several variables, linear transformation, Derivatives is an open subject, Chain rule, Partial derivatives, Jacobian, interchange of the order of differentiation, Derivation of higher order, inverse function theorem, implicit function theorem.

Unit IV **(14 Lecture Hours)**

Riemann Stieltje's Integral: definition and existence of Integral, Properties of integral, integration and differentiation, Riemann sums and Riemann integral, Improper Integrals. Fundamental theorem of Calculus, 1^{st} and 2^{nd} mean value theorems for Riemann Stieltje's integral, Integration of vector valued functions, Rectifiable curves.

Recommended Books:

1. Walter Rudin, Principles of Mathematical Analysis, 3rd edition, McGraw Hill, Kogakusha, 1976, International student edition.
2. H. L. Royden , Real Analysis, 3rd edition, Macmillan, New York & London 1988.
3. Malik, S.C. : Mathematical Analysis, Wiley Eastern Ltd.
4. Titchmarsh, E.C. *The Theory of functions, 2nd Edition*, U.K. Oxford University Press 1961.
5. Tom M. Apostol, Mathematical Analysis , Addition –Wesley.
6. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill Ltd.

Course Title: Symmetries and Differential Equations
Paper Code: MAT.705
Total Lectures: 56

L	T	P	Credits	Marks
4	0	0	4	100

Unit I **(15 Lecture Hours)**

Dimensional Analysis: Buckingham Pi-Theorem, Assumptions Behind Dimensional Analysis, Conclusions from Dimensional Analysis, Proof ofthe Buckingham Pi-Theorem and Examples, Application of Dimensional Analysis to Partial Differential Equations, Generalization of Dimensional Analysis-Invariance of Partial Differential Equations Under Scalings of Variables

Unit II **(14 Lecture Hours)**

Lie Group of transformations: Groups, Examples of Groups, Groups of Transformations, One-Parameter Lie Group of Transformations, Examples of One-Parameter Lie Groups of Transformations, Infinitesimal Transformations: First Fundamental Theorem of Lie, Infinitesimal Generators, Invariant Functions

Unit III **(14 Lecture Hours)**

Canonical Coordinates, Invariant Surfaces, Invariant Curves,Invariant Points
Extended Transformations: Extended Group Transformations-One Dependent and One Independent Variable, Extended Infinitesimal Transformations-One Dependent and One Independent Variable, Extended Transformations-One Dependent and n Independent Variables

Unit IV **(13 Lecture Hours)**

Multi-Parameter Lie Groups of Transformations; Lie Algebras, r-Parameter Lie Groups of Transformations, Lie Algebras, Examples of Lie Algebras, Solvable Lie Algebras

Recommended Books:

1. Bluman GW and Anco SC, Symmetry and Integration Methods for Differential Equations, Appl. Math. Sci., 154, Springer, New York (2002).
2. Bluman GW and Kumei S, Symmetries and Differential Equations, Appl. Math. Sci., Springer-Berlin (1989).
3. Olver PJ, Applications of Lie Groups to Differential Equations, Springer-Verlag (1993).
4. Ovsiannikov LV, Group Properties of Differential Equations, Novosibirsk, Moscow (1962).

Course Title: Fractional Calculus
Paper Code: MAT.706
Total Lectures: 56

L	T	P	Credits	Marks
4	0	0	4	100

Unit I **(15 Lecture Hours)**

Special functions of Fractional Calculus: Gamma Function, Some properties of Gamma function, Beta function, Contour integral representation. Fractional derivatives and integrals, Grunwald Letnikov Fractional derivatives, Riemann-Liouville Fractional Derivatives. Some Other Approaches. Geometric and Physical Interpretation of Fractional Integration and Fractional Differentiation

Unit II **(13 Lecture Hours)**

Sequential Fractional Derivatives. Left and Right Fractional Derivatives. Properties of Fractional Derivatives. Laplace Transforms of Fractional Derivatives. Fourier Transforms of Fractional Derivatives. Mellin Transforms of Fractional Derivatives.

Unit III **(15 Lecture Hours)**

Linear Fractional Differential Equations: Fractional Differential Equation of a General Form. Existence and Uniqueness Theorem as a Method of Solution. Dependence of a Solution on Initial Conditions. The Laplace Transform Method. Standard Fractional Differential Equations. Sequential Fractional Differential Equations.

Unit IV **(13 Lecture Hours)**

Fractional Differential Equations: Introduction, Linearly Independent Solutions, Solutions of the Homogeneous Equations, Solution of the Non-homogeneous Fractional Differential Equations, Reduction of Fractional Differential Equations to ordinary differential equations. Semi Differential equations

Recommended Books:

1. K.B. Oldham & J. Spanier, The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order, Dover Publications Inc, 2006.
2. K.S. Miller & B.Ross., An Introduction to the Fractional Calculus and Fractional Differential Equations Hardcover, Wiley Blackwell, 1993.

Course Title: Advance Theory of Partial Differential Equations
Paper Code: MAT.707
Total Hours: 56

L	T	P	Credits	Marks
4	0	0	4	100

Objectives:
The objective of this course is to equip the students with knowledge of some basic as well as advanced concepts related to partial differential equations and to understand some basic approach to mathematical oriented PDEs.

Unit I **(Lecture Hours 15)**
Distribution-Test Functions and Distributions, Examples, Operations on Distributions, Supports and Singular Supports, Convolution, Fundamental Solutions, Fourier Transform, Schwartz space, Tempered Distributions.
Sobolev spaces-Basic properties, Approximation by smooth functions, Extension theorems, Compactness theorems, Dual spaces, Functional order spaces, Trace spaces, Trace theory, Inclusion theorem.

Unit II **(Lecture Hours 14)**
Weak solutions of Elliptic Boundary Value Problems-Variational problems, Weak formulation of Elliptic PDE, Regularity, Galerkin Method, Maximum principles, Eigenvalue problems, Introduction to finite element methods.

Unit III **(Lecture Hours 13)**
Evolution Equations- Unbounded linear operators, C_0 – Semigroups, Hille-Yosida theorem, Contraction Semigroup on Hilbert Spaces, Heat equation, Wave equation, Schrodinger equation, Inhomogeneous equations.

Unit IV **(Lecture Hours 14)**
Calculus of Variations-Euler-Lagrange Equation, Second variation, Existence of Minimizers(Coactivity, Lower Semi-continuity, Convexity), Regularity, Constraints(Nonlinear Eigenvalue problems, Variational Inequalities, Harmonic maps, Incompressibility), Critical points(Mountain Pass theorem and Applications to Elliptic PDE).

Reference Books:

1. Kesavan, S. *Topics in Functional Analysis and Application.*, New Delhi: Wiley-Eastern, New International, 1999.
2. Evans, L. C. *Partial Differential Equations. Graduate Studies in Mathematics*, Providence: AMS, 1998

Course Title: Advanced Numerical Analysis
Paper Code: MAT.708
Total Hours: 56

L	T	P	Credits	Marks
4	0	0	4	100

Objective:
The aim of this course is to teach the applications of various numerical techniques for a variety of problems occurring in daily life. At the end of the course, the students will be able to do programming in C/C++/MATLAB and understand the basic concepts in Numerical Analysis of differential equations.

Unit-I **(Lecture Hours 15)**
Finite difference approximation to partial derivatives, parabolic equations: An explicit method, Crank Nicolson Implicit method, solution of implicit equations by Gauss Elimination, derivative Boundary conditions, local truncation error, Convergence and stability, Multi-dimensional search without using derivatives, the Method of Rosen brock, Cyclic coordinate method, Method of Hooke and Jeeves and their convergence.

Unit-II **(Lecture Hours 14)**
Hyperbolic equations: Implicit difference methods for wave equation solution of advection equation by finite difference method and Maccormack method, stability analysis, Lax, Wendroff explicit method on rectangular mesh for 1st order equations, Iterative methods for elliptic equations.

Unit-III **(Lecture Hours 14)**
Numerical Differentiation, Trapezoidal and Simpson's one third, Simpson's three eight rule for Numerical integration, adaptive Integration, Boole, Weddle rule, Double integration.
Multidimensional search using derivatives, Steepest Descent algorithm and its convergence analysis, Newton's method and modified Newton's method. Methods using conjugate directions: the method of Davidon-Fletcher- Powell (DFP) method, the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method

Unit-IV **(Lecture Hours 13)**
Constrained optimization: Indirect methods, the concept of penalty functions, exterior penalty function method (EPF), exact absolute value and augmented Lagrangian Penalty methods and their convergence analysis. Direct methods, successive linear programming approximation (SLP), successive quadratic programming approximation (SQP), gradient project method of Rosen, generalized reduced gradient method (GRG), convex simplex algorithm of Zangwill

Reference Books:

1. Smith, G. D. *Numerical solution of Partial Differential Equations: Finite Difference Methods, third edition.* New York, NY: Oxford University Press, 1985.
2. Bradie, B. *A friendly introduction to Numerical Analysis.* Delhi: Pearson Education, 2007.
3. Reddy, J.N. *An Introduction to Finite Element Methods.* Delhi: McGraw-Hill, 2000.Bazaraa, M.S., Sherali, H.D. and Shetty, C.M. *Nonlinear Programming Theory and Algorithms.* Delhi: John Wiley and Sons, 2004.

Course Title: Operational Research
Paper Code: MAT.709

L	T	P	Credits	Marks
4	0	0	4	100

Total Hours: 55
Objective:
The objective of this course is to acquaint the students with the concept of convex sets, their properties and various separation theorems so as to tackle with problems of optimization of functions of several variables over polyhedron and their duals. The results, methods and techniques contained in this paper are very well suited to the realistic problems in almost every area

Unit I **(14 Lecture Hours)**
Operations Research and its Scope. Necessity of Operations Research in industry Mathematical formulation of linear programming problem Linear Programming and examples, Convex Sets, Hyper plane, Open and Closed half-spaces, Feasible, Basic Feasible and Optimal Solutions, Extreme Point & graphical methods. Simple method, Charnes-M method, two phase method, Determination of Optimal solutions, unrestricted variables.

Unit II **(14 Lecture Hours)**
Duality theory, Dual linear Programming Problems, fundamental properties of dual Problems, Complementary slackness, unbounded solution in Primal. Dual Simplex Algorithm, Sensitivity analysis. Integer Programming- Branch and Bound Technique.

Unit III **(14 Lecture Hours)**
The General transportation problem, transportation table, duality in transportation problem, loops in transportation tables, linear programming formulation, solution of transportation problem, test for optimality, degeneracy, transportation algorithm(MODI method), time minimization transportation problem. Assignment Problems: Mathematical formulation of assignment problem, the assignment method, typical assignment problem, the traveling salesman problem.

Unit IV **(14 Lecture Hours)**
Non-Linear Programming: Characteristics, Concepts of convexity, maxima and minima of functions of n-variables using Lagrange multipliers and Kuhn-Tuker conditions, One dimensional search methods, Fibonacci, golden section method and gradient methods for unconstrained problems.
Recommended books:

1. Taha, H. A. Operations Research - An Introduction, New York: Macmillan Publishing Company Inc., 2006.
2. Swarup, K., Gupta, P. K., and Mohan, Man: Operations Research, New Delhi: Sultan Chand & Sons, 2001.
3. Bazaraa, M. S., and Shetty, C. M. Nonlinear Programming, Theory & Algorithms, New York: Wiley, 2004.
4. Sinha, S. M. Mathematical Programming, Theory and Methods, Delhi: Elsevier, 2006.
5. Mangasarian, O. L. Nonlinear Programming, Delhi: TATA McGraw Hill Company Ltd., 1969.
6. Hadley, G. Linear Programming, New Delhi: Narosa Publishing House, 1987.
7. Kambo, N. S. Mathematical Programming Techniques, New Delhi: Affiliated East- West Press Pvt. Ltd., 1984, Revised Edition, Reprint 2005.

Course Title: Linear Algebra
Paper Code: MAT.710
Total Lectures: 56

L	T	P	Credits	Marks
4	0	0	4	100

Objective:
The concepts and techniques from linear algebra are of fundamental importance in many scientific disciplines. The main objective is to introduce basic notions in linear algebra that are often used in mathematics and other sciences. The emphasis will be to combine the abstract concepts with examples in order to intensify the understanding of the subject.

Unit I **(14 Lecture Hours)**
Vector Space: vector spaces, subspaces, direct sum of subspaces, linear dependence and independence, basis and dimensions, linear transformations, quotient spaces, algebra of linear transformations, linear functions, dual spaces, matrix representation of a linear transformation, rank and nullity of a linear transformation, invariant subspaces.

Unit II **(15 Lecture Hours)**
Characteristic polynomial and minimal polynomial of a linear transformation, eigenvalues and eigenvectors of a linear transformation, diagonalization and triangularization of a matrix, Cayley Hamilton Theorem, Matrix representation of Linear Transformation, Change of Basis, Canonical forms, Diagonal forms, triangular forms, Jordan Canonical Forms.

Unit III **(14 Lecture Hours)**
Bilinear forms, symmetric bilinear forms, Sylvester's theorem, quadratic forms, Hermitian forms. Inner product spaces. Norms and Distances, Orthonormal basis, Orthogonality, Schwartz inequality, The Gram-Schmidt Orthogonalization process,

Unit IV **(13 Lecture Hours)**
The Adjoint of a Linear operator on an inner product space, Normal and self-Adjoint Operators, Unitary and Normal Operators, Bilinear and Quadratic forms, reduction and classification of quadratic forms.

Recommended books:

1. P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul, First Course in Linear Algebra (Wiley Eastern , Delhi).
2. J. Gilbert and L. Gilbert: Linear Algebra and Matrix Theory (Academic Press).
3. I.N. Herstein, Topics in Algebra (Delhi Vikas).
4. V.Bist and V. Sahai, Linear Algebra (Narosa, Delhi).
5. K. Hoffman and R. Kunze: Linear Algebra 2nd edition, Pearson 2004

Course Title: Probability Thoery
Paper Code: MAT.711
Total Hours: 56

L	T	P	Credits	Marks
4	0	0	4	100

Objectives:
The course is designed to equip the students with various probability distributions and to develop greater skills and understanding of Sampling and Estimation.

Unit I **(13 Lecture Hours)**
Probability: Definition of probability-classical, relative frequency, statistical and axiomatic approach, Addition theorem, Boole's inequality, Conditional probability and multiplication theorem, Independent events, Mutual and pairwise independence of events, Bayes' theorem and its applications.

Unit II **(14 Lecture Hours)**
Random Variable and Probability Functions: Definition and properties of random variables, discrete and continuous random variables, probability mass and density functions, distribution function.

Concepts of bivariate random variable: joint, marginal and conditional distributions.

Mathematical Expectation: Definition and its properties. Variance, Covariance, Moment generating function- Definitions and their properties. Chebychev's inequality.

Unit III **(14 Lecture Hours)**
Discrete distributions: Uniform, Bernoulli, binomial, Poisson and geometric distributions with their properties.

Continuous distributions: Uniform, Exponential, Gamma and Normal distributions with their properties. Central Limit Theorem (Statement only).
Statistical estimation: Parameter and statistic, sampling distribution and standard error of estimate. Point and interval estimation, Unbiasedness, Efficiency.

Unit IV **(15 Lecture Hours)**
Sampling Theory: Types of Sampling, errors in sampling, Parameter and Statistic, Tests of Significance: Null Hypothesis, Alternative Hypothesis, One-tailed, Two-tailed tests. sampling Attributes: Tests of Significance for single proportion and difference of proportions. Sampling of Variables.

Recommended books:

1. Mood, A.M., Graybill, F.A. and Boes, D.C., Mc Graw Hill Book Company.
2. Freund,J.E., Mathematical Statistics, Prentice Hall of India.
3. Gupta S.C. and Kapoor V.K., Fundamentals of Mathematical Statistics, S. Chand Pub., New Delhi.
4. Speigel, M., Probability and Statistics, Schaum Outline Series.
5. Ross, Sheldon M. (2003) Introductory Statistics
6. Hogg, R. V. and Craig, T. T. (1978) Introduction to Mathematical Statistics (Fourth Edition) (Collier-McMillan)

Course Title: Number Theory
Paper Code: MAT.712
Total Hours: 56

L	T	P	Credits	Marks
4	0	0	4	100

Objective:
The objectives of this course is to teach the fundamentals of different branches of Number Theory, namely, Geometry of Numbers and Analytic Number Theory.

Unit I **(14 Lecture Hours)**
Divisibility of Integers, Greatest common divisor, Euclidean algorithm. The Fundamental theorem of Arithmetic, Congruences, Residue classes and reduced residue classes.

Unit II **(14 Lecture Hours)**
Chinese remainder theorem, Fermat's little theorem, Wilson's theorem, Euler's theorem. Arithmetic functions σ(n), d(n), τ(n), µ(n), Order of an integer modulo n, primitive roots for primes, composite numbers having primitive roots, theory of indices.

Unit III **(14 Lecture Hours)**
Quadratic residues, Legendre symbol, Euler's criterion, Gauss's lemma, Quadratic reciprocity law, Jacobi symbol. Perfect numbers, Characterization of even perfect numbers, Elementary results on the distribution of primes, Twin primes, Mersenne primes and Fermat numbers.

Unit IV **(14 Lecture Hours)**
Representation of an integer as a sum of two and four squares. Diophantine equations ax +by =c, x2+y2=z2, x4+y4=z4. Farey sequences, continued Fractions, Farey dissection of a circle and its applications to approximations of irrationals by rationals. Finite and Infinite simple continued fractions, periodic and purely periodic continued fractions, Lagrange's Theorem on periodic continued fractions. Applications to Pell's equation. The fundamental solution of Pell's equation.

Recommended books:

1. David, M. Burton Elementary Number Theory, 7th Edition New Delhi: Tata McGraw-Hill 2012.
2. Niven, I., Zuckeman, S. and Montgomery, H.L. Introduction to Number Theory, Wiley Eastern 1991.
3. Apostal, T.N. Introduction to Analytic Number Theory, Springer Verlag 1976.
4. Hardy, G.H. and Wright, E. M. An Introduction to the Theory of Number, U.K.:Oxford Univ. Press 2008.

Course Title: Advanced Algebra
Paper Code: MAT.713
Total Hours: 56

L	T	P	Credits	Marks
4	0	0	4	100

Objective:
This course is a basic course in Algebra for students who wish to pursue research work in Algebra. Contents have been designed in accordance with the UGC syllabi in mind.

Unit I **(13 Lecture Hours)**
Field Theory: Basic concepts of Field theory, Extension of fields, algebraic and transcendental extensions. Splitting fields, Separable and inseparable extensions, Algebraically closed fields, Perfect fields.

Unit II **(14 Lecture Hours)**
Galios Theory: Galois extensions, the fundamental theorem of Galois theory, Cyclotomic extensions, and Cyclic extensions, Applications of cyclotomic extensions and Galois theory to the constructability of regular polygons, Solvability of polynomials by radicals.

Unit III **(15 Lecture Hours)**
Modules: Difference between Modules and Vector Spaces, Module Homomorphisms, Quotient Module, Completely reducible or Semi simple Modules, Free Modules, Representation and Rank of Linear Mappings, Smith normal Form over a PID, Finitely generated modules over a PID, Rational Canonical Form, Applications to finitely generated abelian groups.

Unit IV **(14 Lecture Hours)**
Canonical forms: Similarity of linear transformations, Invariant subspaces, Reduction to triangular form, Nilpotent transformations, Index of nilpotency, Invariants of nilpotent transformations, The primary decomposition theorem, Rational canonical forms, Jordan blocks and Jordan forms.

Recommended Books:

1. First Course in Linear Algebra, P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul, (Wiley Eastern ,Delhi).
2. Linear Algebra and Matrix Theory, J. Gilbert and L. Gilbert, (Academic Press).
3. Topics in Algebra, I.N. Herstein, (Delhi Vikas).
4. V.Bist and V. Sahai, Linear Algebra (Narosa, Delhi).
5. J-P. Escofier, Galois Theory, Springer-Verlag.
6. I. Stewart, Galois Theory, Chapman and Hall.
7. Hartley, B and Hawkes T.O., Rings, Modules and Linear Algebra, Chapman and Hall.

Musili C, Rings and Modules (Second Revised Edition), Narosa Publishing House, New Delhi, 1994.

Course Title: Functional Analysis
Paper Code: MAT.714
Total Hours: 56

L	T	P	Credits	Marks
4	0	0	4	100

Objective:

The objective of this course is to introduce basic concepts, methods of Functional Analysis and its Applications. It is a first level course in Functional Analysis.

Unit I **(13 Lecture Hours)**

Fundamentals of Normed Linear spaces: Normed Linear spaces, Banach spaces and examples, Characterization of finite dimensional spaces , Quotient space of normed linear spaces and its completeness, Equivalent norms, Riesz Lemma, Basic properties of finite dimensional normed linear spaces and compactness.

Unit II **(14 Lecture Hours)**

Weak convergence and bounded linear transformations, Normed linear spaces of bounded linear transformations, Dual spaces with examples. **Three main theorems on Banach space:** Uniform boundedness theorem and some of its consequences, Open mapping and closed graph theorems.

Unit III **(14 Lecture Hours)**

Hahn-Banach theorem for real linear spaces, Complex linear spaces and normed linear spaces, Reflexive spaces, Weak sequential compactness, Compact operators, Solvability of linear equations in Banach spaces, the closed Range Theorem.

Unit IV **(15 Lecture Hours)**

Inner product spaces. Hilbert spaces, Orthonormal Sets, Bessel's Inequality, Complete orthonormal sets and Parseval's Identity, Structure of Hilbert Spaces, Projection Theorem, Riesz Representation Theroem, Adjoint of an operator on a Hilbert space, Reflexivity of Hilbert Spaces, Self-adjoint operators, Positive, projection, normal and Unitary operators, Abstract variational boundary-value problem, The Generalized Lax-Milgram Theorem.

Recommended books:

1. Limaye, B. V. *Functional Analysis,* New Delhi: New Age International (P) Ltd, 1996.
2. Berberian, S. K. *Introduction to Hilbert Spaces,* Rhode Island: AMS Chelsea Publishing, 1996.
3. Goffman, C., and Pedrick, G. *First Course in Functional Analysis*, New Delhi: Prentice Hall of India, 1983.
4. Riesz, F. K., and Nagy, B. S. *Functional Analysis*, Dover Publications, 1990.
5. Siddiqui, A. H. *Functional Analysis*, New Delhi: Tata-McGraw Hill, 1987.
6. Erwin Kreyszig, Introductory functional analysis with application, Willey.

Course Title: Complex Analysis
Paper Code: MAT.715
Total Lectures: 54

L	T	P	Credits	Marks
4	0	0	4	100

Objective:
The objective of the course is to provide foundation for other related branches of Mathematics. Most of the topics covered are widely applicable in Applied Mathematics and Engineering.

Unit I **(13 Lecture Hours)**
Review of Complex number system, Algebra of complex numbers, the complex plane, Function of a complex variable, Limit, Continuity, Uniform continuity, Differentiability, Analytic function, Cauchy- Riemann equations, Harmonic functions and Harmonic conjugate, Construction of analytic functions.

Unit II **(14 Lecture Hours)**
Complex line integral, Cauchy’s theorem, Cauchy-Goursat theorem, Cauchy’s integral formula and its generalized form. Index of a point with respect to a closed curve, Cauchy’s inequality. Poisson’s integral formula, Morera’s theorem. Liouville’s theorem, Contour integral, Power series, Taylor’s series, Higher order derivatives, Laurent’s series.

Unit III **(14 Lecture Hours)**
Singularities of analytic functions, Casorati-Weierstrass theorem, Fundamental theorem of algebra, Zeroes of analytic function, Poles, Residues, Residue theorem and its applications to contour integrals, Branches of many valued functions with arg z, log z, and z^{a}.Maximum modulus principle, Schwarz lemma, Open mapping theorem.

Unit IV **(13 Lecture Hours)**
Meromorphic functions, The argument principle, Rouche’s theorem, Mobius transformations and their properties and classification, Definition and examples of conformal mappings.

Recommended books:

1. E. T. Copson, An Introduction to Theory of Functions of a Complex variable
2. L. V. Ahlfors, Complex Analysis, Tata McGraw Hill, 1979.
3. S. Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 2007.
4. R. V. Churchill & J. W. Brown, Complex Variables and Applications, Tata McGraw Hill, 1996.
5. W. Tutschke and H.L. Vasudeva, An Introduction to complex analysis: Classical and Modern Approaches, CRC Publications.

Course Title: Differential Equations and Boundary-Value Problems
Paper Code: MAT.716
Total Lectures: 55

L	T	P	Credits	Marks
4	0	0	4	100

Unit-I **(15 Lecture Hours)**
Existence and uniqueness of solutions of ODEs, power series solution, singular points, some special functions. Nonlinear system of ODE : Preliminary concepts and definitions, the fundamental existence-uniqueness results, dependence on initial conditions and parameters, the maximum interval of existence,

Unit-II **(13 Lecture Hours)**
linearlization, stability and Liapunov functions, saddle, nodes, foci and centers, normal form theory and Hamiltonian systems. Boundary value problems : Green's function method, Sturm-Liouville problem.

Unit-III **(13 Lecture Hours)**
First-order PDEs, Cauchy problem, method of characteristics, Second-order PDEs, classification, characteristics and canonical forms. Elliptic boundary value problems : Maximum principle, Green's function,

Unit-IV **(14 Lecture Hours)**
Sobolev spaces, variational formulations, weak solutions, Lax-Milgram theorem, trace theorem, Poincarénequality, energy estimates, Fredholm alternative, regularity estimates, system of conservation laws, entropy criteria.

Recommended Books:

1. L. Perko, Differential Equations and Dynamical Systems, Springer, 2001.
2. J. Guckenheimer, P. Holmes, Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields, Springer-Verlag, New York, 1983.
3. S. Wiggins, Introduction to Applied Nonlinear Dynamical Systems and Chaos, Springer-Verlag, New York, 1990
4. Lawrence C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 19, American Mathematical Society, Providence, 1998
5. Robert C. McOwen, Partial Differential Equations - Methods and Applications, Pearson Education Inc., Indian Reprint 2004.
6. S.J. Farlow, Partial Differential Equations for Scientists and Engineers, Dover Publications,New York, 1982.

Course Title: Potential Flow of Fluids and Water-wave Theory
Paper Code: MAT.717
Total Lectures: 56

L	T	P	Credits	Marks
4	0	0	4	100

Unit I **(15 Lecture Hours)**

Real fluids and ideal fluids, velocity of fluid at a point, streamlines, path lines, streak lines, velocity potential, vorticity vector, local and particle rate of change, equation of continuity, irrigational and rotational motion, acceleration of fluid, conditions at rigid boundary.

Unit II **(13 Lecture Hours)**

Euler's equation of motion, Bernoulli's equation, their applications, Potential theorems, axially symmetric flows, impulsive motion, Kelvin's Theorem of circulation, equation of vorticity.

Unit III **(15 Lecture Hours)**

Equations of Motion. Two dimensional flow. Navier-Stokes equation of motion. Velocity potential and Laplace equation. Simple irrotational flows. Separation of variables for an axisymmetric flow. Bernoulli equation for unsteady irrotational flow. Deep water wave. Shallow water wave.

Unit IV **(13 Lecture Hours)**

Theory of surface wave. Finite amplitude wave. One dimensional tidal dynamics. Linear and non-linear diffraction theory. Permutation methods. Water wave interaction with submerged spherical structures and floating cylinders. Solitary waves. Cnoidal wave. Schrodinger equation.

Recommended Books:

1. G.K. Batchelor, An Introduction to Fluid Dynamics, CUP.
2. O.M. Phillips, The Dynamics of Upper Ocean, CUP
3. J.J. Stoker, Water Waves, Interscience.

Course Title: Stochastic Processes and Queueing Theory
Paper Code: MAT.718
Total Lectures: 56

L	T	P	Credits	Marks
4	0	0	4	100

Unit I **(13 Lecture Hours)**
Review of probability, random variables and distributions, generating functions and transforms; Stochastic processes, discrete and continuous-time Markov chains, renewal processes,

Unit II **(14 Lecture Hours)**
Brownian motion; Characteristics of queueing systems, Little's formula, Markovian and non-Markovian queueing systems, embedded Markov chain applications to M/G/1, G/M/1, and related queueing systems,

Unit III **(14 Lecture Hours)**
Queues with vacations, priority queues, queues with modulated arrival process, discrete-time queues, and matrix-geometric methods in queues; Networks of queues, open and closed queueing networks,

Unit IV **(13 Lecture Hours)**
Algorithms to compute the performance metrics; Simulation of queues and queueing networks; Application to manufacturing, computer and communication systems and networks.

Recommended Books:

1. L. Kleinrock, Queueing Systems, Vol. 1: Theory, 1975, Vol. 2: Computer Applications, 1976, John Wiley and Sons.
2. J. Medhi, Stochastic Models in Queueing Theory, 2nd Edition, Academic Press, 2002.
3. S. Asmussen, Applied Probability and Queues, 2nd Edition, Springer, 2003.
4. D. Gross, and C.Harris, Fundamentals of Queueing Theory, 3rd Edition, John Wiley and Sons, 1998.
5. R.B. Cooper, Introduction to Queueing Theory, 2nd Edition, North-Holland, 1981.
6. R. Nelson, Probability, Stochastic Processes, and Queueing Theory: The Mathematics of Computer Performance Modelling, Springer-Verlag, 1995.
7. E. Gelenbe, and G. Pujolle, Introduction to Queueing Networks, 2nd Edition, John Wiley, 1998.

Course Title: Reliability Theory
Paper Code: MAT.719
Total Lectures: 56

L	T	P	Credits	Marks
4	0	0	4	100

Unit I **(14 Lecture Hours)**
Reliability concepts and measures:Components and systems, coherent systems, reliability of coherent systems, cuts and paths, modular decomposition, bounds on system reliability, structural and reliability importance of components.

Unit II **(14 Lecture Hours)**
Life distributions and associated survival, conditional survival and hazard rate functions. Exponential, Weibull, gamma life distributions and estimation of their parameters.

Unit III **(14 Lecture Hours)**
Notions of ageing. IFR IFRA, NBU, DMRL, NBUE, and HNBUE classes; their duals and relationships between them. Closures of these classes under formation of coherent systems, convolutions and mixtures.

Unit IV **(14 Lecture Hours)**
Partial orderings: Convex, star, stochastic, failure rate and mean-residual life orderings. Univariate shock models and life distributions arising out of them. Maintenance and replacement policies, availability of repairable systems.

Recommended Books:

1. 1Barlow R.E. and Proschan F. (1985), Statistical Theory of Reliability and Life Testing; Holt, Rinehart and Winston.
2. Lawless J.F. (1982), Statistical Models and Methods of Life Time Data; John Wiley Models; Marcel Dekker.
3. Shaked M. and Shanthikumar J.G. (1994), Stochastic Orders & Their Applications, Academic Press.

Course Title: Sampling Theory
Paper Code: MAT.720
Total Hours: 52

L	T	P	Credits	Marks
4	0	0	4	100

Objectives:
The course is designed to equip the students with basic knowledge of different sampling schemes, their mean and variance estimations and non-sampling errors.

Unit I (13 Lecture Hours)
Basic ideas and distinctive features of sampling; Probability sampling designs, sampling schemes, inclusion probabilities and estimation; Review of important results in simple and stratified random sampling.

Unit II (15 Lecture Hours)
Sampling with varying probabilities (unequal probability sampling): PPSWR /WOR methods [cumulative total and Lahiri's scheme] and related estimators of a finite population total or mean (Hansen – Hurwitz and Des Raj estimators for a general sample size and Murthy's estimator for a sample of size 2). Horvitz – Thompson Estimator (HTE) of a finite population total /mean. Non-negative variance estimation. Ratio and Regression Estimators.

Unit III (15 Lecture Hours)
Double (two-phase) sampling with special reference to the selection with unequal probabilities in at least one of the phases; Double sampling for ratio and regression estimators of population mean, systematic sampling and its application to structured populations; Cluster sampling- equal clusters; Two-stage sampling with equal size of first stage units.

Unit IV (9 Lecture Hours)
Non-sampling error with special reference to non-response problems. Small Area Estimation.

Recommended books:

1. Chaudhuri, A. (2010). Essentials of Survey Sampling. Prentice Hall of India.
2. Chaudhari, A. and Stenger, H. (2005). Survey Sampling Theory and methods, 2ndEdn., Chapman and Hall.
3. Cochran, W.G. (1977). Sampling Techniques, John Wiley & Sons, New York
4. Hedayat, A.S., and Sinha, B.K. (1991). Design and Inference in Finite Population Sampling, Wiley, New York.
5. Levy, P.S. and Lemeshow, S. (2008). Sampling of Populations-Methods and Applications, Wiley.
6. Thompson, Steven K. (2002). Sampling, John Wiley and Sons, New York.