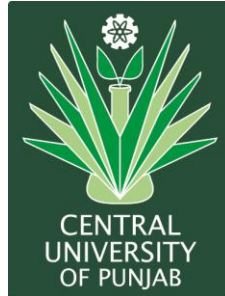


Centre for Mathematics and Statistics, CUPB

**Central University of Punjab, Bathinda**



**Course Scheme & Syllabus**

**for**

**M.Sc. MATHEMATICS**



# Centre for Mathematics and Statistics, CUPB

## Scheme of Programme for M.Sc. Mathematics

### SEMESTER- I

S. No	Course Code	Course Title	Course Type	L	T	P	Cr	% Weightage				$T_M$
								$C_A$	$M_1$	$M_2$	$E_T$	
1	MAT.506	Probability and Distribution Theory	C	4	-	-	4	25	25	25	25	100
2	MAT.507	Real Analysis	C	4	-	-	4	25	25	25	25	100
3	MAT.508	Topology	C	4	-	-	4	25	25	25	25	100
4	MAT.509	Linear Algebra	C	4	-	-	4	25	25	25	25	100
5	MAT.510	Differential Equations	C	4	-	-	4	25	25	25	25	100
6	XXX	Inter-Disciplinary Elective -1 (From Other Departments)	$I_E$	2	-	-	2	25	25	25	25	50
Interdisciplinary courses offered by Mathematics Faculty (For PG students of other Centres)												
7	MAT.503	Basic Mathematics	$I_E$	2	-	-	2	25	25	25	25	50
				<b>22</b>	-	-	<b>22</b>	-	-	-	-	<b>550</b>

$C_A$  : Continuous Assessment: Based on Objective Type Tests/ Assignments

$M_1$  : Mid-Term Test-1: Based on Objective Type & Subjective Type Test

$M_2$  : Mid-Term Test-2: Based on Objective Type & Subjective Type Test

$E_T$  : End-Term Exam (Final): Based on Objective Type Tests

$T_M$  : Total Marks

C: Core;  $I_E$ : Interdisciplinary elective; F: Foundation; L: Lectures; T: Tutorial; P: Practical; Cr: Credits.

## SEMESTER- II

S. No	Course Code	Course Title	Course type	L	T	P	Cr	% Weightage				
								$C_A$	$M_1$	$M_2$	$E_T$	$T_M$
1	MAT.521	Computer Fundamentals and C Programming	F	3	0	0	3	25	25	25	25	75
2	MAT.522	Computer Fundamentals and C Programming (Lab)	F	0	0	2	1	-	-	-	-	25
3	MAT.523	Algebra – I	C	4	-	-	4	25	25	25	25	100
4	MAT.524	Measure Theory	C	4	-	-	4	25	25	25	25	100
5	MAT.525	Differential Geometry of Curves and Surfaces	C	4	-	-	4	25	25	25	25	100
6	MAT.526	Complex Analysis	C	4	-	-	4	25	25	25	25	100
7	MAT.527	Mechanics	C	2	-	-	2	25	25	25	25	50
8	XXX	Humanities for Science Students (From Other Departments)	$I_E$	2	-	-	2	25	25	25	25	50
Interdisciplinary course offered by Mathematics faculty for PG students other centres												
9	MAT.504	Linear Programming	$I_E$	2	-	-	2	25	25	25	25	50
	MAT.505	Numerical Methods		2	-	-	2	25	25	25	25	50
				<b>22</b>	<b>-</b>	<b>4</b>	<b>24</b>	<b>-</b>	<b>-</b>	<b>-</b>	<b>-</b>	<b>600</b>

$C_A$  : Continuous Assessment: Based on Objective Type Tests/ Assignments

$M_1$  : Mid-Term Test-1: Based on Objective Type & Subjective Type Test

$M_2$  : Mid-Term Test-2: Based on Objective Type & Subjective Type Test

$E_T$  : End-Term Exam (Final): Based on Objective Type Tests

$T_M$  : Total Marks

C: Core;  $I_E$  : Interdisciplinary elective; F: Foundation; L: Lectures; T: Tutorial; P: Practical; Cr: Credits.

## Semester-III

S. No	Course Code	Course Title	Course Type	L	T	P	Cr	% Weightage				$T_M$
								$C_A$	$M_1$	$M_2$	$E_T$	
1	MAT.502	Research Methodology	F	2	0	0	2	25	25	25	25	50
2	MAT.551	Algebra-II	C	4	-	-	4	25	25	25	25	100
3	MAT.552	Calculus of Variation and Integral Equation	C	4	-	-	4	25	25	25	25	100
4	MAT.553	Numerical Analysis	C	3	-	-	3	25	25	25	25	75
5	MAT.554	Numerical Analysis (Lab)	C	-	-	2	1	-	-	-	-	25
6	MAT.597	Seminar	F	-	-	-	2	-	-	-	-	50
7	MAT.555	Differential Topology	E	4	-	-	4	25	25	25	25	100
	MAT.556	Advanced Complex Analysis										
	MAT.557	Advanced Partial Differential Equations										
8	MAT.558	Discrete Mathematics	E	4	-	-	4	25	25	25	25	100
	MAT.559	Number Theory										
	MAT.560	Operations Research										
				<b>23</b>	<b>-</b>	<b>2</b>	<b>24</b>	<b>-</b>	<b>-</b>	<b>-</b>	<b>-</b>	<b>600</b>

$C_A$  : Continuous Assessment: Based on Objective Type Tests// Assignments

$M_1$  : Mid-Term Test-1: Based on Objective Type & Subjective Type Test

$M_2$  : Mid-Term Test-2: Based on Objective Type & Subjective Type Test

$E_T$  : End-Term Exam (Final): Based on Objective Type Tests

$T_M$  : Total Marks

C: Core; E: Elective; F: Foundation; L: Lectures; T: Tutorial; P: Practical; Cr: Credits.

## Semester-IV

S. No	Course Code	Course Title	Course Type	L	T	P	Cr	% Weightage				$T_M$
								$C_A$	$M_1$	$M_2$	$E_T$	
1	MAT.571	Functional Analysis	C	4	-	-	4	25	25	25	25	100
2	MAT.599	Project Work	C		-	16	8	-	-	-	-	200
3	MAT.572	Riemannian Geometry	E	4	-	-	4	25	25	25	25	100
	MAT.573	Fluid Mechanics										
	MAT.574	Advanced Numerical Analysis										
4	MAT.575	Algebraic Topology	E	4	-	-	4	25	25	25	25	100
	MAT.576	Lie Groups and Lie Algebra										
	MAT.577	Finite Element Analysis										
				<b>12</b>		<b>16</b>	<b>20</b>	-	-	-	-	<b>500</b>

$C_A$  : Continuous Assessment: Based on Objective Type Tests/ Assignments

$M_1$  : Mid-Term Test-1: Based on Objective Type & Subjective Type Test

$M_2$  : Mid-Term Test-2: Based on Objective Type & Subjective Type Test

$E_T$  : End-Term Exam (Final): Based on Objective Type Tests

$T_M$  : Total Marks

C: Core; E: Elective; F: Foundation; L: Lectures; T: Tutorial; P: Practical; Cr: Credits.

Semester-I**Course Title: Probability and Distribution Theory****Course Code: MAT.506****Total Hours: 60****Objectives:**

The course is designed to equip the students with knowledge of various probability distributions and to develop greater skills and understanding of various inequalities for further studies.

L	T	P	Credits	Marks
4	0	0	4	100

**Unit I****(14 Lecture Hours)**

Random experiments, sample spaces (finite and infinite), events, algebra of events, three basic approaches to probability, combinatorial problems. Axiomatic approach to probability. Product sample spaces, conditional probability, Bayes' formula.

**Unit II****(16 Lecture Hours)**

Bernoulli trials, random variables (discrete and continuous). Distribution Function and its properties, mean and variance. Discrete Distributions: Bernoulli, binomial, Poisson, hypergeometric, geometric, negative binomial, uniform. Continuous Distributions: Uniform, normal, exponential, gamma, Beta, Cauchy, Weibull, Pareto, Laplace and Lognormal.

**Unit III****(15 Lecture Hours)**

Bivariate random variable and their joint, marginal and conditional p.m.f.s. and p.d.f.s, correlation coefficient, conditional expectation. Bivariate normal distributions. Moment generating and probability generating functions. Functions of random variables and their distributions using Jacobian of transformation and other tools. Probability Integral transformation, order statistics and their distributions (continuous case only), truncated distributions, compound distributions.

**Unit IV****(14 Lecture Hours)**

Markov's, Chebychev's, Holder's, Jensen's and Liapounov's inequalities. Convergence in probability and in distribution, Weak law of large numbers. Central limit problem; De-Moivre-Laplace and Lindberg-Levy forms of central limit theorem. Approximating distribution of a function of a statistic (Delta method). Transformation of statistics.

**Recommended Books:**

1. V. K. Rohtagi and A. K. M. E. Saleh, *An Introduction to Probability Theory and Mathematical Statistics*, 3<sup>rd</sup> Edition, Wiley, 2015.
2. I. Miller and M. Miller, *Mathematical Statistics*, 6<sup>th</sup> Edition, Oxford & IBH Pub., 1999.
3. S. M. Ross, *Introduction to Probability Models*, 11<sup>th</sup> Edition, Academic Press, 2014.

**Suggested Readings:**

1. E. J. Dudewicz and S. N. Mishra, *Modern Mathematical Statistics*, Wiley International Student Edition, 1988.
2. P. Billingsley, *Probability and Measure*, 4<sup>th</sup> Edition, John Wiley & Sons, 2012.

**Course Title:** Real Analysis

**Course Code:** MAT.507

**Total Lectures:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objective:** The aim of this course is to make the students learn fundamental concepts of metric spaces, Riemann-Stieltjes integral as a generalization of Riemann Integral, Sequence and series of functions and some basic theorems.

### Unit-I

(15 Lecture Hours)

**Set Theory:** Finite, countable and uncountable sets

**Metric spaces:** Definition and examples, Open and closed sets, Compact sets, Elementary properties of compact sets,  $k$ - cells, Compactness of  $k$ -cells, Compact subsets of Euclidean space  $\mathcal{R}^k$ , Perfect sets, Cantor set, Separated sets, Connected sets in a metric space, Connected subsets of real line.

### Unit-II

(15 Lecture Hours)

**Sequences in Metric spaces:** Convergent sequences, Subsequences, Cauchy sequences, Complete metric space, Cantor's intersection theorem, Category of a set and Baire's category theorem. Examples of complete metric space, Banach contraction principle.

### Unit-III

(15 Lecture Hours)

**Continuity:** Limits of functions (in Metric spaces), Continuous functions, Continuity and compactness, Continuity and connectedness, Discontinuities, Monotonic functions, Uniform continuity.

**Riemann Stieltje's Integral:** Definition and existence of Riemann Stieltje's integral, Properties of integral. Integration and Differentiation. Fundamental Theorem of Calculus, 1st and 2nd Mean Value Theorems of Riemann Stieltje's integral.

### Unit-IV

(15 Lecture Hours)

**Sequences and series of functions:** Problem of interchange of limit processes for sequences of functions, Uniform convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, equicontinuous families of functions, Stone Weierstrass Theorem.

### Recommended Books:

1. Walter Rudin, *Principles of Mathematical Analysis*, 3rd Edition, McGraw Hill, Kogakusha, International student Edition, 1976.
2. S. C. Malik, *Mathematical Analysis*, Wiley Eastern Ltd., 2010.

### Suggested Readings:

1. E. C. Titchmarsh, *The Theory of functions*, 2nd Edition, Oxford University Press 1961.
2. Tom M. Apostol, *Mathematical Analysis*, Addition –Wesley, 2002.
3. Ajit Kumar and S. Kumaresan, *A Basic Course in Real Analysis*, Narosa, Publishing House, 2014.
4. R. G. Bartle, *The Elements of Real Analysis*, John Willey and Sons, 1976.



**Course Title:** Topology

**Course Code:** MAT.508

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objective:** The course is an introductory course on point-set topology. It is designed in such a way that the students will have a working knowledge in general topology and be able to understand more advanced topics like Algebraic Topology, Differential Topology, Riemannian Geometry and allied areas.

**Unit-I**

**(16 Lecture Hours)**

Topological Spaces: Open sets, Closed sets, Neighborhoods, Bases, Sub bases, Limit points, Closures, Interiors, Continuous functions, Homeomorphisms. Examples of topological spaces: Subspace topology, Product topology, Metric topology, Topological manifolds. Quotient Topology: Construction of cylinder, Cone, Mobius band and Torus.

**Unit-II**

**(15 Lecture Hours)**

Connected spaces, Connected subspaces of the real line, Components and path components, Local connectedness. Compact spaces, Sequentially compact spaces, Heine-Borel Theorem, Compact subspaces of the real line, Limit point compactness, Local  $\omega$ -compactness and one point compactification.

**Unit-III**

**(15 Lecture Hours)**

The Countability Axioms: Separable spaces, Lindelof spaces. Separation Axioms: Hausdorff spaces, Regularity, Complete regularity, Normality, Urysohn Lemma, Urysohn Metrization Theorem, Tietze Extension Theorem and Tychonoff Theorem.

**Unit-IV**

**(14 Lecture Hours)**

The Stone- $\hat{C}$ ech Compactification, Complete metric spaces, Compactness in metric spaces, Pointwise and compact convergence, Ascoli's Theorem and Baire spaces.

**Recommended Books:**

1. J. R. Munkres, *Topology- A First Course*, Prentice Hall of India, New Delhi, 1975.
2. M. A. Armstrong, *Basic Topology*, Springer, Paperback Edition, 2004.
3. S. Kumaresan, *Topology of Metric Spaces*, second edition, Narosa Publishing House New Delhi, 2015.

**Suggested Readings:**

1. K. D. Joshi, *Introduction to General Topology*, Wiley Eastern, Delhi, 1986.
2. M. G. Murdeshwar, *General Topology*, Wiley Eastern, New Delhi, 1983.
3. G. F. Simmons, *Introduction to Topology & Modern Analysis*, McGraw Hill, Auckland, 1963.
4. James Dugundji, *Topology*, Universal Book Stall, New Delhi, 1990.
5. S. Willord, *General Topology*, Philippines: Addison Wesley Publishing Company, 1970.

**Course Title:** Linear Algebra

**Course Code:** MAT.509

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objective:**

The concepts and techniques from linear algebra are of fundamental importance in many scientific disciplines. The main objective is to introduce basic notions in linear algebra that are often used in mathematics and other sciences. The emphasis will be to combine the abstract concepts with examples in order to intensify the understanding of the subject.

**Unit I**

**(15 Lecture Hours)**

**Vector Space:** Vector spaces, Subspaces, Direct sum of subspaces, Linear dependence and independence, Basis and dimensions, Linear transformations, Algebra of linear transformations, Matrix representation of a linear transformation, Rank and nullity of a linear transformation, Invariant subspaces. Change of basis,

**Unit I**

**(16 Lecture Hours)**

Characteristic polynomial and minimal polynomial of a linear transformation, Cayley Hamilton theorem, Eigenvalues and eigenvectors of a linear transformation, Diagonalization and triangularization of a matrix, Characteristic polynomial and minimal polynomial of block matrices. Canonical forms, Diagonal forms, Triangular forms, Jordan canonical forms, rational canonical forms, Quotient spaces.

**Unit III**

**(15 Lecture Hours)**

Linear functional, Dual space, Dual basis, Annihilators, Bilinear forms, Symmetric bilinear forms, Sylvester's theorem, quadratic forms, Hermitian forms. Reduction and classification of quadratic forms.

**Unit IV**

**(14 Lecture Hours)**

Inner product spaces. Norms and distances, Orthonormal basis, Orthogonality, Schwartz inequality, The Gram-Schmidt orthogonalization process. Orthogonal and positive definite matrices. The Adjoint of a linear operator on an inner product space, Normal and self-adjoint operators, Unitary and orthogonal operators,

**Recommended Books:**

1. J. Gilbert and L. Gilbert, *Linear Algebra and Matrix Theory*, Cengage Learning, 2004.
2. V. Bist and V. Sahai, *Linear Algebra*, Narosa, Delhi, 2002.

**Suggested Readings:**

1. I. N. Herstein, *Topics in Algebra* 2<sup>nd</sup> Edition, Wiley Eastern Limited, New Delhi, 2006.
2. K. Hoffman and R. Kunze: *Linear Algebra* 2<sup>nd</sup> Edition, Pearson Education (Asia) Pvt. Ltd/ Prentice Hall of India, 2004.
3. P. B. Bhattacharya, S.K. Jain and S.R. Nagpaul, *First Course in Linear Algebra*, Wiley Eastern, Delhi, 2003.

**Course Title:** Differential Equations

**Course Code:** MAT.510

**Total Hours:** 60

**Objective:**

The objective of this course is to equip the students with knowledge of some advanced concepts related to differential equations and to understand some basic approaches to solve the ordinary and partial differential equation.

L	T	P	Credits	Marks
4	0	0	4	100

**Unit-I**

**(15 Lecture Hours)**

Initial value problem, Boundary Value Problems, Lipchitz's condition, Dependence of solution on initial conditions and on function. Existence and Uniqueness theorem (Picard's Method), Non local existence of solutions.

**General theory of homogenous and non-homogeneous linear ODEs:** Solution of Linear Homogeneous Equations; Wronskian and Linear Independence, Reduction of the order of equation, Non-Homogeneous equations: Method of undetermined coefficients, Variation of parameters.

**Unit-II**

**(14 Lecture Hours)**

**Series Solutions of Second Order Linear Equations:** Ordinary points, Regular and Irregular Singular points of second order linear ODEs, Power series solution near an ordinary point, Cauchy-Euler Equations, Solutions about Singular Points; The Method of Frobenius

**Unit-III**

**(14 Lecture Hours)**

Total differential equations, Simultaneous differential equations, Adjoint and self adjoint equations, Green's function and its applications to boundary value problems, Sturm Liouville's boundary value problems. Sturm comparison and separation theorems, Orthogonal solutions.

**Unit-IV**

**(17 Lecture Hours)**

Classification of Partial differential Equations (PDEs), Cauchy's problem and Characteristics for first order PDEs, Lagrange's linear PDEs, Charpit's and Jacobi's method. Well posed and Ill-posed problems, General solution of higher order linear PDEs with constant coefficients, Separation of variables method for Laplace, Heat and wave equations.

**Recommended Books:**

1. L. C. Evans, *Partial Differential Equations. Graduate Studies in Mathematics*, American Mathematical Society, 2<sup>nd</sup> Edition, Indian Reprint, 2014.
2. I. N. Sneddon, *Elements of Partial Differential Equations*, McGraw-Hill, 2006.
3. S. L. Ross, *Differential Equations*, Wiley, 1984.
4. M. D. Raisinghania, *Advanced Differential Equations*, S. Chand & Company Ltd., New Delhi, 2001.

**Suggested Readings:**

1. E. A. Coddington and N. Levinson, *Theory of Ordinary Differential Equations*, McGraw Hill, New York, 1955.
2. E. B. Williams and C. DiPrima Richard, *Elementary Differential Equations and Boundary Value Problems*, John Wiley and Sons, New York, 1967.
3. W.T. Reid, *Ordinary Differential Equations*, John Wiley and Sons, New York, 1971.

# Centre for Mathematics and Statistics, CUPB

10

**Course Title:** Basic Mathematics

**Course Code:** MAT.503

**Total Hours:** 30

L	T	P	Credits	Marks
2	0	0	2	50

**Objective:** The objective of this course is to provide the understanding of basic mathematical techniques for the post graduate students of the other departments.

## Unit-I

(08 Lecture Hours)

**Sets:** Basic Definitions, subsets, power set, set operations. Ordered pairs, Cartesian product of sets.

**Functions and Relations:** Definition of relation, domain, co-domain and range of a relation. Binary relations, equivalence relations, partition. Function as a special kind of relation from one set to another. Domain, co-domain and range of a function. composition, inverse. Real valued function of the real variable, constant, identity, Polynomial, rational, Functions.

## Unit-II

(07 Lecture Hours)

Sequence and series, Arithmetic Progression (A.P), Arithmetic Mean (A.M), Geometric Progression (G.P), general term of a G.P, sum of n terms of a G.P. Arithmetic and Geometric series, infinite G.P. and its sum. Geometric mean (G.M), relation between A.M and G.M.

## Unit-III

(08 Lecture Hours)

Need for complex numbers, especially  $\sqrt{-1}$ , to be motivated by inability to solve every Quadratic equation. Brief description of algebraic properties of complex numbers. Argand plane and polar representation of complex numbers, Statement of Fundamental Theorem of Algebra,  $n^{\text{th}}$  roots of unity.

## Unit-IV

(07 Lecture Hours)

Matrices and types of matrices, Operations on Matrices, Determinants of Matrix and Properties of Determinants, Minors and Cofactor and Adjoint of a square matrix, Singular and non-singular Matrices, Inverse of a Matrix, Eigen values and Eigen vectors, Cayley Hamilton theorem.

## Recommended Books:

1. R.K. Jain and S.R.K. Iyengar, *Advanced Engineering Mathematics*, Narosa Publications, 8<sup>th</sup> Edition, 2002.
2. G. B. Thomas and R. L. Finney, *Calculus and Analytic Geometry*, 11<sup>th</sup> edition, Pearson India, 2015.

## Suggested Reading Books:

1. E. Kreyszig, *Advanced Engineering Mathematics*, 9<sup>th</sup> edition, John Wiley & Sons, Inc., 2006.
2. P. K. Jain, *Mathematics: Text book for class XI*, NCERT, 2006.

**Semester-II**

**Course Title:** Computer Fundamentals and C Programming

**Course Code:** MAT.521

**Total Hours:** 45

L	T	P	Credits	Marks
3	0	0	3	75

**Objectives:** The aim of this course is to provide adequate knowledge of fundamentals of computer along with problem solving techniques using C programming. This course provides the knowledge of writing modular, efficient and readable C programs. Students also learn the utilization of arrays, structures, functions, pointers, file handling and their applications.

**Unit-I**

**(10 Lecture Hours)**

**Computer Hardware:** Definitions, Historical overview, Technological advancement in computers, Shape of today's computer, Computer as a system. CPU, Primary memory, Secondary storage devices, Input and Output devices,

**Unit-II**

**(11 Lecture Hours)**

**Computer Software:** Significance of software in computer system, Categories of software – System software, Application software, Compiler, Interpreter, Utility program, Binary arithmetic for integer and fractional numbers, Operating System and its significance.  
Introduction to algorithm, Flow charts, Problem solving methods, Need of programming languages.

**Unit-III**

**(12 Lecture Hours)**

**C Programming:** Historical development of C, C character set, Identifiers and keywords, Data types, Declarations, Statement and symbolic constants, Input-output statements, Preprocessor commands, Operators, Expressions, Library functions, Decision making and loop control statements

**Unit-IV**

**(12 Lecture Hours)**

**C Programming:** Functions, Storage Classes, Arrays, Strings, Pointers, Structure and Union, File handling.

**Recommended Books:**

1. P. Norton, *Introduction to Computers*, Tata McGraw Hill, 2008.
2. B. W. Kerninghan and D.M. Ritchie, *The C Programming Language*, 2<sup>nd</sup> Edition, PHI, New Delhi, 2011.

**Suggested Readings:**

1. Y. Kanetkar, *Let Us C*, 13<sup>th</sup> Edition, BPB Publications, 2013.
2. V. Rajaraman, *Fundamentals of Computers*, PHI, 2004.
3. G.B. Shelly, T.J. Cashman and M.E. Vermaat, *Introduction to Computers*, Cengage India Pvt Ltd, 2008.

## Centre for Mathematics and Statistics, CUPB

12

**Course Title:** Computer Fundamentals and  
C Programming (LAB)

**Course Code:** MAT.522

**Total Hours:** 30

Laboratory experiments will be set in context with the materials covered in the theory.

L	T	P	Credits	Marks
0	0	2	1	25

# Centre for Mathematics and Statistics, CUPB

13

**Course Title:** Algebra – I

**Course Code:** MAT.523

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

## Objective:

This course provides the foundation required for more advanced studies in Algebra. The aim is also to develop necessary prerequisites for course Algebra-II.

## Unit I

(15 Lecture Hours)

**Group Theory:** Review of basic concepts of Groups, Subgroups, Normal subgroups, Quotient groups, Homomorphism, Cyclic groups, Permutation groups, Even and odd permutations, Conjugacy classes of permutations, Alternating groups, Cayley's Theorem, Class equations.

## Unit II

(15 Lecture Hours)

Normal and Subnormal series, Composition series, Solvable groups, Nilpotent groups. Direct products, Fundamental theorem for finite Abelian groups, Sylow theorems and their applications, Survey of some finite groups, Groups of order  $p^2$ ,  $pq$  ( $p$  and  $q$  primes)

## Unit III

(14 Lecture Hours)

**Ring theory:** Review of rings, Elementary properties of Rings, Zero Divisors, Nilpotent and idempotent elements, Characteristic of rings, Ideals, Ring homomorphism, Maximal and prime ideals, Nilpotent and nil ideals, Zorn's Lemma.

## Unit IV

(16 Lecture Hours)

Polynomial rings in many variables, Factorization of polynomials in one variable over a field. Unique factorization Domains. Euclidean and Principal ideal Domains. Gauss lemma, Eisenstein's irreducibility criterion, Unique factorization in  $R[x]$ , where  $R$  is a Unique factorization domain.

## Recommended Books:

1. J. A. Gallian, *Contemporary Abstract Algebra*, Narosa Publishing House, New Delhi, 2008.
2. I. N. Herstein, *Topics in Algebra*, 2<sup>nd</sup> Edition, Wiley Eastern Limited, New Delhi, 2006.
3. P. B. Bhattacharya, S.K. Jain and S.R Nagpal, *Basic Abstract Algebra*, Cambridge University Press, New Delhi, 2003.

## Suggested Readings:

1. T. W. Hungerford, *Algebra*, Springer, 1974.
2. M. Artin, *Algebra*, (Second Edition) Prentice Hall of India, 2011.  
S. Surjeet and Q. Zameeruddin, *Modern Algebra*, 8<sup>th</sup> Edition, Vikas Publishing House, New Delhi, 2006.

# Centre for Mathematics and Statistics, CUPB

14

**Course Title:** Measure Theory

**Course Code:** MAT.524

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objective:** The objective of this course is to introduce student's measure theory in an abstract setting after having studied Lebesgue measure on real line. Some important theorems are also studied.

## Unit-I

(15 Lecture Hours)

Semi-algebras, Algebras, Monotone class,  $\sigma$ -algebras, Measure and outer measures, Caratheodory extension process of extending a measure on semi-algebra to generated  $\sigma$ -algebra, Completion of a measure space.

## Unit-II

(15 Lecture Hours)

Borel sets, Lebesgue outer measure and Lebesgue measure on  $\mathbb{R}$ , Translation invariance of Lebesgue measure, Characterizations of Lebesgue measurable sets, Countable additivity, Continuity of measure and Borel-Cantelli Lemma, Existence of a non-measurable set, Measurability of Cantor set.

## Unit-III

(15 Lecture Hours)

Measurable functions on a measure space and their properties, Borel and Lebesgue measurable functions, Simple functions and their integrals, Littlewood's three principle and Egoroff's Theorem (statement only), Lebesgue integral on  $\mathbb{R}$  and its properties.

## Unit-IV

(15 Lecture Hours)

Bounded convergence theorem, Fatou's lemma, Lebesgue monotone convergence theorem, Lebesgue dominated convergence theorem,  $L^p$  spaces, Young's inequality, Minkowski's and Hölder's inequalities, Riesz-Fischer theorem (statement only).

## Recommended Books:

1. H.L. Royden, *Real Analysis*, Macmillan, New York, 1988.
2. G.de Bara, *Measure Theory and Integration*, Ellis Horwood Limited, England, 2003.
3. P. R. Halmos, *Measure Theory, Grand Text Mathematics*, 14<sup>th</sup> Edition, Springer, 1994.

## Suggested Readings:

1. I. K. Rana, *An Introduction to Measure and Integration*, 2<sup>nd</sup> Edition, Narosa Publishing House, New Delhi, 2005.
2. B. Krishna and A. Lahiri, *Measure Theory*, Hindustan Book Agency, 2006.
3. Terence Tao, *An Introduction To Measure Theory*, American Mathematical Society, 2012.
4. G.B. Folland, *Real Analysis*, 2<sup>nd</sup> Edition, John Wiley, New York, 1999.



# Centre for Mathematics and Statistics, CUPB

15

**Course Title:** Differential Geometry of Curves and Surfaces

**Course Code:** MAT.525

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objective:** : To introduce students to the local and global theory of curves and surfaces so that they can embark on further studies and research in topics like Differential Topology, Riemannian Geometry and allied areas.

## Unit-I

(15 Lecture Hours)

Curves in Plane and Space: Parameterized curves, Tangent vector, Arc length, Reparametrization, Regular curves, Curvature and Torsion of smooth curves, Frenet-Serret formulae, arbitrary speed curves, Frenet approximation of a space curve. Osculating plane, Osculating circle, Osculating sphere, Involutives and Evolutes, Bertrand curves, Spherical indicatrices, Helices, Fundamental theorem of space curves.

## Unit-II

(15 Lecture Hours)

Isomerisms of  $R^3$ , Congruence of curves. Surfaces in  $R^3$ : Definition and examples, Smooth surfaces, Tangent, Normal and Orientability. Examples of surfaces: Generalized cylinder and generalized cone, Ruled surfaces, Surface of revolution and Quadric surfaces. First fundamental form, Isometries of surfaces, Conformal mapping of surfaces, Surface Area, Equi-area maps and Theorem of Archimedes,

## Unit-III

(15 Lecture Hours)

Second fundamental form, Curvature of curves on a surface, Normal and Principal curvatures, Meusnier's theorem, Euler's theorem, Weingarten equations and Weingarten matrix, Geometric interpretation of principal curvatures, Umbilical points. Gaussian and Mean curvature, Pseudo sphere, Flat surfaces, Surfaces of constant mean curvature, Gaussian curvature of compact surfaces, Gauss map and its properties.

## Unit-IV

(15 Lecture Hours)

Geodesics: Definition and basic properties, Geodesic equations, Geodesics on a surfaces of revolution, Clairaut's theorem, Geodesics as shortest paths, Geodesic coordinates, Gauss Theorema Egregium, Gauss equations, Codazzi-Mainardi equations, Compact surfaces of constant Gaussian curvature.

## Recommended Books:

1. A. Pressley, *Elementary Differential Geometry*, Springer (Undergraduate Mathematics Series), 2001.
2. M. P. Do Carmo, *Differential Geometry of Curves and Surfaces*, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1976.
3. B. O' Neill, *Elementary Differential Geometry*, Academic Press, 1997.

## Suggested Readings:

1. C. Bär, *Elementary Differential Geometry*, Cambridge University Press, 2001.
2. A. Gray, *Differential Geometry of Curves and Surfaces*, CRC Press, 1998.
3. R. S. Millman & G. D. Prkar, *Elements of Differential Geometry*, Prentice Hall, Englewood, Clifts, NJ, 1977.
4. T. J. Willmore, *An Introduction to Differential Geometry*, Oxford University Press, London, 1965.

# Centre for Mathematics and Statistics, CUPB

16

**Course Title:** Complex Analysis

**Course Code:** MAT.526

**Total Lectures:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objective:** This course is aimed to provide an introduction to the theories for functions of a complex variable. It begins with the exploration of the algebraic, geometric and topological structures of the complex number field. The concepts of analyticity, Cauchy-Riemann equations and harmonic functions are then introduced. Students will be equipped with the understanding of the fundamental concepts of complex variable theory.

## **Unit-I** (15 Lecture Hours)

Review of complex number system, Algebra of complex numbers, Complex plane, Function of a complex variable, Limit, Continuity, Uniform continuity, Differentiability, Analytic function, Cauchy- Riemann equations, Harmonic functions and Harmonic conjugate.

## **Unit-II** (15 Lecture Hours)

Complex line integral, Cauchy's theorem, Cauchy-Goursat theorem, Cauchy's integral formula and its generalized form, Index of a point with respect to a closed curve, Cauchy's inequality. Poisson's integral formula, Morera's theorem. Liouville's theorem, Contour integral, Power series, Taylor's series, Higher order derivatives, Laurent's series.

## **Unit-III** (15 Lecture Hours)

Singularities of analytic functions, Fundamental theorem of algebra, Zeroes of analytic function, Poles, Residues, Residue theorem and its applications to contour integrals, Branches of many valued functions with  $\arg z$ ,  $\log z$ , and  $z^a$ . Maximum modulus principle, Schwarz lemma, Open mapping theorem.

## **Unit-IV** (15 Lecture Hours)

Meromorphic functions, The argument principle, Rouché's theorem, Möbius transformations and their properties and classification, Definition and examples of conformal mappings.

### **Recommended Books:**

1. L. V. Ahlfors, *Complex Analysis*, Tata McGraw Hill, 1979.
2. S. Ponnusamy, *Foundations of Complex Analysis*, Narosa Publishing House, 2007.

### **Suggested Readings:**

1. W. Tutschke and H.L. Vasudeva, *An Introduction to Complex Analysis: Classical and Modern Approaches*, CRC Publications, 2004.
2. R. V. Churchill & J. W. Brown, *Complex Variables and Applications*, Tata McGraw Hill, 1996.

# Centre for Mathematics and Statistics, CUPB

17

**Course Title:** Mechanics

**Course Code:** MAT.527

**Total Hours:** 30

L	T	P	Credits	Marks
2	0	0	2	50

## Objectives:

This course is designed for the M.Sc. students, but it is also useful for science or engineering students in related areas. The main goal of the course is to introduce the concept of mechanics and its applications and to learn the fundamentals of this important topic.

## Unit-I

**(08 Lecture Hours)**

General force system, equipollent force system, equilibrium conditions, reduction of force systems, couples, moments and wrenches, necessary and sufficient conditions of rigid bodies, general motion of rigid body.

## Unit-II

**(07 Lecture Hours)**

Moments and products of inertia and their properties, Moving frames of references and frames in general motion, Euler's dynamical equations, Motion of a rigid body with a fixed point under no force.

## Unit-III

**(08 Lecture Hours)**

Method of point set constraints, generalized coordinates D'Alembert's principle and Lagrange's equations, Applications of Lagrangian formulation. Hamilton's principle, Techniques of calculus of variations.

## Unit-IV

**(07 Lecture Hours)**

Lagrange's equations through Hamilton's principle, Cyclic coordinates and conservation theorems, Canonical equations of Hamilton, Hamilton's equations from variational principle.

## Recommended Books:

1. K. Sankra Rao, *Classical Mechanics*, Prentice Hall of India, 2005.
2. M.R. Speigal, *Theoretical Mechanics*, Schaum Outline Series, 1967
3. N.C. Rana and P.S. Joag, *Classical Mechanics*, Tata McGraw- Hill, New Delhi, 1991.

## Suggested Readings:

1. F. Gantmacher, *Lectures in Analytic Mechanics*, MIR Publishers, Moscow, 1975.
2. P.V. Panat, *Classical Mechanics*, Narosa Publishing House, New Delhi, 2005.
- 3.
4. Louis N. Hand and Janet D. Finch, *Analytical Mechanics*, Cambridge University Press, 1998.
5. D.E Rutherford, *Classical Mechanics*, Oliver & Boyd Ltd., 3<sup>rd</sup> Edition, 1964

# Centre for Mathematics and Statistics, CUPB

18

**Course Title:** Linear Programming

**Course Code:** MAT.504

L	T	P	Credits	Marks
2	0	0	2	50

**Objective:** The objective of this course is to provide the understanding of Linear Programming for the post graduate students of the other departments.

## Unit-I

**(08 Lecture Hours)**

Formulation of linear programming problems (LPP). Graphical solution to LPPs. Cases of unique and multiple optimal solutions. Unbounded solutions and infeasibility and redundant constraints.

## Unit-II

**(08 Lecture Hours)**

Feasible solution, basic feasible solutions, Optimal solution, Convex sets, Solution of LPP with Simplex methods. The dual problem. Formulation of the dual.

## Unit-III

**(08 Lecture Hours)**

Transportation problems, Formulation of transportation problem, Feasible and optimal solution of transportation problems. Assignment problems.

## Unit-IV

**(06 Lecture Hours)**

Theory of games: Introduction to basic concepts of game theory including strategic Games.

## Recommended Books:

1. H. A. Taha, *Operations Research - An Introduction*, Macmillan Publishing Company Inc., New York, 2006.
2. K . Swarup, P. K. Gupta and Man Mohan, *Operations Research*, Sultan Chand & Sons, New Delhi, 2001.

# Centre for Mathematics and Statistics, CUPB

19

**Course Title:** Numerical Methods

**Course Code:** MAT.505

**Total Lectures:** 30

L	T	P	Credits	Marks
2	0	0	2	50

**Objective:** The objective of this course is to provide the understanding and use of numerical methods for the post graduate students of other departments.

## Unit-I

**(07 Lecture Hours)**

Error Analysis: Relative error, Truncation error, Roundoff error, Order of approximation, Order of convergence, Propagation.

## Unit-II

**(08 Lecture Hours)**

Bisection method, Secant method, Newton Raphson method, Convergence and order of convergence.

## Unit-III

**(07 Lecture Hours)**

Interpolation and Polynomial Approximation, Lagrange's Method, Newton's polynomials.

## Unit-IV

**(08 Lecture Hours)**

Solution of Differential Equations: Euler's method, Heun's method, Taylor series method, Runge Kutta method.

## Recommended Books:

1. C. F. Gerald and P. O. Wheatly, *Applied Numerical Analysis*, 7<sup>th</sup> Edition, Pearson LPE, 2009.
2. R. S. Gupta, *Elements of Numerical Analysis*, Cambridge University Press, 2<sup>nd</sup> Edition, 2015.
3. M. K. Jain, S.R.K. Iyengar and R.K. Jain, *Numerical Methods for Scientific and Engineering Computation*, 6<sup>th</sup> Edition, New Age International, New Delhi, 2015.

## Suggested Reading:

1. K. Atkinson, *An Introduction to Numerical Analysis*, John Wiley & Sons, 2<sup>nd</sup> Edition, 2012.
2. J. I. Buchaman and P. R. Turner, *Numerical Methods and Analysis*, Prentice-Hall, 1988.
3. S. S. Sastry, *Introduction Methods of Numerical Analysis*, Prentice-Hall, 4<sup>th</sup> Edition, 2005.

# Centre for Mathematics and Statistics, CUPB

20

## Semester-III

**Course Title:** Research Methodology

**Course Code:** MAT.502

**Total Hours:** 30

### **Objectives:**

The objective of this course is to equip the students with knowledge of some basic as well as advanced concepts related to research. The course covers preparation of research plan, reading and understanding of scientific papers, scientific writing, research proposal writing, ethics, plagiarism etc.

L	T	P	Credits	Marks
2	0	0	2	50

### **Unit-I**

[ 7 Hours]

**Introduction:** Meaning, Objectives, Characteristics, Significance, and Types of Research; Research Approaches, Research Methods vs. Research Methodology, Research Process, and Criteria of Good Research.

### **Unit-II**

[ 8 Hours]

**Literature Survey and Review:** Meaning of Literature Survey and Review, Sources of Literature, Methods of Literature Review, and Techniques of Writing the Reviewed Literature. **Formulating Research Problem:** Understanding a Research Problem, Selecting the Research Problem, Steps in Formulation of a Research Problem, Formulation of Research Objectives, and Construction of Hypothesis.

### **Unit-III**

[ 7 Hours]

**Research Design:** Meaning of and Need for Research Design, Characteristics of a Good Research Design, Different Research Designs, Basic Principles of Experimental Designs, Data Collection, Processing, and Interpretation.

### **Unit-IV**

[ 8 Hours]

**Report Writing:** Types of Reports – Technical and Popular Reports, Significance of Report Writing, Different Steps in Writing Report, Art of Writing Research Proposals, Research Papers, Project Reports, and Dissertations/Thesis; Basics of Citation and Bibliography/Reference Preparation Styles; Report Presentation: Oral and Poster Presentations of Research Reports.

### **Recommended Books:**

1. Kothari, C.R. and G. Garg (2014): *Research Methodology: Methods and Techniques*, 3<sup>rd</sup> ed., New Age International Pvt. Ltd. Publisher
2. Kumar, R. (2014): *Research Methodology – A Step-By-Step Guide for Beginners*, 4<sup>th</sup> ed., Sage Publications

### **Suggested Readings:**

1. Anderson, J. (2001): *Thesis and Assignment Writing*, 4<sup>th</sup> ed., Wiley, USA
2. Dawson, Catherine, (2014): *Practical Research Methods*, New Delhi, UBS Publishers' Distributors.
3. Gray, David E. (2004): *Doing Research in the Real World*. London, UK: Sage Publications.

**Course Title:** Algebra–II

**Course Code:** MAT.551

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objective:**

This course is an advanced course in Algebra for students who wish to pursue research work in Algebra.

**Unit-I**

**(15 Lecture Hours)**

**Field Theory:** Basic concepts of field theory, Extension of fields, algebraic and transcendental extensions. Algebraically closed fields, Splitting fields, Separable and inseparable extensions, Normal extension, Multiple roots, Finite fields, Perfect fields.

**Unit-II**

**(15 Lecture Hours)**

**Modules:** Definition and Examples, Submodules, Direct sum of submodules, Free modules, Difference between modules and vector spaces, Quotient modules, Homomorphism, Simple modules, Modules over PID

**Unit-III**

**(14 Lecture Hours)**

**Modules with chain conditions:** Artinian Modules, Noetherian Modules, composition series of a module, Length of a module, Hilbert Basis Theorem.

**Unit-IV**

**(16 Lecture Hours)**

**Galois Theory:** Automorphism groups, Fixed fields, Galois extensions, The fundamental theorem of Galois theory, Cyclotomic extensions, and Cyclic extensions, Applications of cyclotomic extensions and Galois theory to the constructability of regular polygons, Solvability of polynomials by radicals.

**Recommended Books:**

1. P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul, *First Course in Linear Algebra*, Wiley Eastern, Delhi, 2008.
2. M. Artin, *Algebra*, 2<sup>nd</sup> Edition, Prentice Hall of India, 2011.
3. D. S. Dummit and R. M. Foote, *Abstract Algebra*, 3<sup>rd</sup> Edition, John Wiley, 2011.

**Suggested Readings:**

1. J. Gilbert and L. Gilbert, *Linear Algebra and Matrix Theory*, Academic Press, 2004.
2. I. N. Herstein, *Topics in Algebra*, 2<sup>nd</sup> Edition, Wiley Eastern Limited, New Delhi, 2006
3. V. Bist and V. Sahai, *Linear Algebra*, Narosa, Delhi, 2002.
4. J. P. Escofier, *Galois Theory*, Springer-Verlag, 2000.
5. I. Stewart, *Galois Theory*, Chapman and Hall, 2003.
6. B. Hartley and T. O. Hawkes, *Rings, Modules and Linear Algebra*, Chapman and Hall, 1970.
7. C. Musili, *Rings and Modules*, 2<sup>nd</sup> Revised Edition, Narosa Publishing House, New Delhi, 1994.

# Centre for Mathematics and Statistics, CUPB

22

**Course Title:** Calculus of Variation and Integral Equations

**Course Code:** MAT.552

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

## Objective:

The objectives of the course calculus of variations and integral equations is to develop knowledge of the basic tenets of the theory of integral equations and mastery of the respective solutions of problems and exercises, knowledge of the main provisions of the calculus of variations and the ability to use the concepts and methods of the theory in solving problems arising in theoretical and mathematical physics.

## Unit-I

(15 Lecture Hours)

Functional, variation of functional and its properties, fundamental lemma of calculus of variation, Euler's - Lagrange's equations of single independent and single dependent variable and application. Necessary and Sufficient Conditions for Extrema. Brachistochrone problem, Functional involving higher order derivatives.

## Unit-II

(15 Lecture Hours)

Sturm-Liouville's theorem on extremals, one sided variations, Hamilton's principle, Hamilton's canonical equation of motion, The principle of least action, Langrange's equations from Hamilton's principle. Variational Methods (Direct Methods, Euler's finite difference method, The Ritz method, Kantorovich Method), for Boundary value problems in ODE's & PDE's, Isoperimetric Problems.

## Unit-III

(15 Lecture Hours)

Volterra Equations: Integral equations and algebraic system of linear equations. Volterra equation L2 kernels and functions. Volterra equations of first and second kind. Volterra integral equation and linear differential equation.

## Unit-IV

(15 Lecture Hours)

Fredholm Equations: Solution by the method of successive approximations. Solution of Fredholm integral equation for degenerate kernel, Solution by the successive approximations, Neumann series and resolvent kernel.

## Recommended books:

1. R. P. Kanwal, *Linear Integral Equations*, Birkhauser, Boston, 1996.
2. A. Pinckus, and S. Zafrany, *Fourier series and Integral Transform*, Cambridge University Press, New York, 1997.

## Suggested Readings:

1. J. L. Synge and B.A. Griffith, *Principle of Mechanics*, McGraw-Hill Book Company, 1970.
2. H. Goldstein, *Classical Mechanics*, Narosa Publishing House, 2<sup>nd</sup> Edition, 1980.
3. Zafar Ahsan, *Lecture Notes on Mechanics, Department of Mathematics*, AMU, 1999.
4. R. P. Kanwal, *Linear Integral Equations*, Birkhauser, Boston, 1996.
5. A. Pinckus, and S. Zafrany, *Fourier series and Integral Transform*, Cambridge University Press, New York, 1997.



**Course Title:** Numerical Analysis

**Course Code:** MAT.553

**Total Hours:** 45

L	T	P	Credits	Marks
3	0	0	3	75

**Objective:**

The aim of this course is to teach the applications of various numerical techniques for a variety of problems occurring in daily life. At the end of the course, the students will be able to do programming in C/C++/MATLAB and understand the basic concepts in Numerical Analysis of differential equations.

**Unit-I (11 Lecture Hours)**

**Error Analysis:** Definition and sources of errors, Propagation of errors, Sensitivity and conditioning, Stability and accuracy, Floating-point arithmetic and rounding errors.  
**Numerical Solutions of Algebraic Equations:** Bisection method. Fixed-point iteration, Newton's method, Secant method, Convergence and order of convergence

**Unit-II (12 Lecture Hours)**

**Linear Systems of Equations:** Gauss Elimination, Gauss-Jordan method, LU decomposition, Gauss-Seidel iteration method.  
**Polynomial Interpolation:** Interpolating polynomial, Lagrange and Newton divided difference interpolation, Error in interpolation, Finite difference formulas, Hermite Interpolation.

**Unit-III (11 Lecture Hours)**

**Spline and Approximation:** Cubic Spline, Least square method, Pade approximation  
**Eigen Value Problems:** Power method.  
**Numerical Differentiation and Integration:** Numerical differentiation with finite differences, Trapezoidal rule, Simpson's 1/3 - rule, Simpson's 3/8 rule, Error estimates for Trapezoidal rule and Simpson's rule, Gauss quadrature formulas.

**Unit-IV (11 Lecture Hours)**

**Numerical Solution of Ordinary Differential Equations:** Solution by Taylor series, Picard Method of successive approximations, Euler's Method, Modified Euler Method, Runge-Kutta Methods. Finite difference method for boundary value problems.

**Recommended Books:**

1. M. K. Jain, S.R.K. Iyengar and R.K. Jain, *Numerical Methods for Scientific and Engineering Computation*, 6<sup>th</sup> Edition, New Age International, New Delhi, 2015.
2. R.L. Burden and J. D. Faires, *Numerical Analysis*, 9<sup>th</sup> Edition, Cengage Learning, 2011.

**Suggested Readings:**

1. S. S. Sastry, *Introductory Methods of Numerical Analysis*, 4<sup>th</sup> Edition, PHI, 2015.
2. C. F. Gerald and P. O. Wheatly, *Applied Numerical Analysis*, 7<sup>th</sup> Edition, Pearson LPE, 2009.
3. R. S. Gupta, *Elements of Numerical Analysis*, Cambridge University Press, 2<sup>nd</sup> Edition, 2015.
4. K. Atkinson, *An Introduction to Numerical Analysis*, John Wiley & Sons, 2<sup>nd</sup> Edition, 1989.

## Centre for Mathematics and Statistics, CUPB

24

**Course Title:** Numerical Analysis (Lab)

**Course Code:** MAT.554

**Total Hours:** 30

L	T	P	Credits	Marks
0	0	2	1	25

**Objective:** Laboratory experiments will be set in context with the materials covered in theory in C/C++/MATLAB.

**Laboratory Work:** Programming exercises on numerical methods using C/C++/MATLAB languages.

1. To detect the interval(s) which contain(s) root of equation  $f(x)=0$  and implement bisection method to find root of  $f(x)=0$  in the detected interval.
2. To find the root of  $f(x)=0$  using Newton-Raphson and fixed point iteration methods.
3. To compute the intermediate value using the Newton's forward difference interpolation formula.
4. To compute Lagrange and divided difference interpolating polynomials.
5. To solve linear system of equations using Gauss elimination (without pivoting) method.
6. To solve linear system of equations using Gauss-Seidel method.
7. To find the dominant eigenvalues and associated eigenvector by Rayleigh power method.
8. To integrate a function numerically using trapezoidal and Simpson's rule.
9. To solve the initial value problem using Euler and modified Euler's methods.
10. To solve the initial value problem using Runge-Kutta methods.

# Centre for Mathematics and Statistics, CUPB

25

**Course Title:** Seminar  
**Course Code:** MAT.597  
**Total Hours:** 60

<b>L</b>	<b>T</b>	<b>P</b>	<b>Credits</b>	<b>Marks</b>
0	0	4	2	50

**Course Title:** Differential Topology

**Paper Code:** MAT.555

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objective:** To introduce students to the basics of Differential Topology so that they are able to appreciate better the topics covered in allied courses like Differential Geometry of Curves and Surfaces and Riemannian Geometry, as well as be adequately prepared for pursuing research in these topics.

**UNIT-I**

**(15 Lecture Hours)**

Topological Manifolds, Charts, Atlases, Smooth Manifolds, Examples of Smooth Manifolds, Smooth functions on a Manifold, Smooth maps between Manifolds, Diffeomorphism, Smoothness in terms of components, Examples of Smooth maps, Partial derivatives, and the Inverse function Theorem.

**UNIT-II**

**(15 Lecture Hours)**

Tangent space, The Differential of a map, Chain rule, Bases for the Tangent space at a point, Curves in a manifold, Immersions and Submersions, Critical and Regular points, Submanifolds, Rank of a smooth map, Immersion and Submersion Theorems, Tangent bundle, Bump functions and partition of unity, Vector fields and Lie bracket.

**UNIT-III**

**(15 Lecture Hours)**

Topological Groups, Lie Groups: Definition and examples, The product of two Lie Groups, Lie subgroups, One parameter subgroups and exponential map, Homomorphism and isomorphism in Lie Groups, Lie transformation Groups, The tangent space and Left invariant vector fields of a Lie Group.

**UNIT-IV**

**(15 Lecture Hours)**

Differential forms, Cotangent spaces, pullback of 1-forms, k-forms, Exterior Product, Differential forms on a circle, Exterior derivative, Exterior algebra and Lie derivative, Global formulas for the Lie and Exterior derivatives.

**Recommended Books:**

1. L. W. Tu, *An Introduction to Manifolds*, Second edition, Springer, 2011.
2. S. Kumaresan, *A Course in Differential Geometry and Lie Groups (Texts and Readings in Mathematics)*, Hindustan Book Agency, 2002.
3. S. S. Chern, W. H. Chen and K. S. Lam, *Lectures on Differential Geometry*, World Scientific Publishing Co. Pte. Ltd., 2000.

**Suggested Readings:**

4. W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian geometry*, 2<sup>nd</sup> edition, Academic Press, New York, 2003.
5. N. J. Hicks, *Notes of Differential Geometry*, D. Van Nostrand Reinhold Company, New York, 1965.
6. L. Conlon, *Differentiable Manifolds*, 2<sup>nd</sup> edition Birkhauser Boston, Cambridge, MA, 2001.
7. J. M. Lee, *Introduction to Smooth Manifolds*, GTM, Vol. 218, Springer, New York, 2003.
8. F. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, Springer, New York, 1983.

**Course Title:** Advanced Partial Differential Equations

**Course Code:** MAT.556

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objectives:**

The objective of this course is to equip the students with knowledge of some advanced concepts related to partial differential equations and to understand some basic approaches to mathematical oriented PDEs.

**Unit-I** **(16 Lecture Hours)**

**Distribution:** Test Functions and Distributions, Examples, Operations on Distributions, Supports and Singular Supports, Convolution, Fundamental Solutions, Fourier Transform, Schwartz space, Tempered Distributions.

**Sobolev Spaces:** Basic properties, Approximation by smooth functions, Extension theorems, Compactness theorems, Dual spaces, Functional order spaces, Trace spaces, Trace theory, Inclusion theorem.

**Unit-II** **(15 Lecture Hours)**

**Weak Solutions of Elliptic Boundary Value Problems:** Variational problems, Weak formulation of Elliptic PDE, Regularity, Galerkin Method, Maximum principles, Eigenvalue problems, Introduction to finite element methods.

**Unit-III** **(14 Lecture Hours)**

**Evolution Equations:** Unbounded linear operators,  $C_0$  – Semigroups, Hille-Yosida theorem, Contraction Semigroup on Hilbert Spaces, Heat equation, Wave equation, Schrodinger equation, Inhomogeneous equations.

**Unit-IV** **(15 Lecture Hours)**

**Calculus of Variations:** Euler-Lagrange Equation, Second variation, Existence of Minimizers (Coactivity, Lower Semi-continuity, Convexity), Regularity, Constraints (Nonlinear Eigenvalue problems, Variational Inequalities, Harmonic maps, Incompressibility), Critical points (Mountain Pass theorem and Applications to Elliptic PDE).

**Recommended Books:**

1. S. Kesavan, *Topics in Functional Analysis and Application*, Wiley-Eastern, New International, New Delhi, 1999.
2. L. C. Evans, *Partial Differential Equations. Graduate Studies in Mathematics*, American Mathematical Society, 2<sup>nd</sup> Edition, Indian Reprint, 2014.

**Course Title:** Advanced Complex Analysis

**Course Code:** MAT.557

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objectives:**

This course is designed to enable the readers to understand further deeper topics of Complex Analysis and will provide basic topics needed for students to pursue research in pure Mathematics.

**Unit-I**

**(16 Lecture Hours)**

**Harmonic function:** Definition, Relation between a harmonic function and an analytic function, Examples, Harmonic Conjugate of a harmonic function, Poisson's Integral formula, Mean Value Property, The maximum & minimum principles for harmonic functions, Dirichlet Problem for a disc and uniqueness of its solution, Characterization of harmonic functions by mean value property.

**Unit-II**

**(16 Lecture Hours)**

**Analytic continuation:** Direct Analytic continuation, Analytic continuations along arcs, Homotopic curves, The Monodromy theorem, Analytic continuation via reflection. Harneck's principle. Open mapping theorem, normal families, The Riemann Mapping Theorem, Picard's theorem.

**Unit-III**

**(14 Lecture Hours)**

**Weierstrass Elliptic functions:** Periodic functions, Simply periodic functions, fundamental period, Jacobi's first and second question, Doubly periodic functions, Elliptic functions, Pair of Primitive Periods, Congruent points, First and Second Liouville's Theorem, Relation between zeros and poles of an elliptic function, Definition of Weierstrass elliptic function  $(z)$  and their properties, The differential equation satisfied by  $(z)$  [i.e., the relation between  $(z)$  and  $(z)$ ], Integral formula for  $(z)$ , Addition theorem and Duplication formula for  $(z)$ .

**Unit- IV**

**(14 Lecture Hours)**

**Weierstrass Zeta function:** Weierstrass Zeta function and their properties, Quasi periodicity of  $(z)$ , Weierstrass sigma function  $(z)$  and their properties, Quasiperiodicity of  $(z)$ , Associated sigma functions.

**Recommended Books:**

1. J. B. Conway, *Functions of One Complex Variable*, Springer-Verlag International, USA, 1978.
2. L.V. Ahlfors, *Complex Analysis: An Introduction to the Theory of Analytic Functions of One Complex Variable*, McGraw-Hill Higher Education, New Delhi, 1979.

**Suggested Readings:**

1. S. Lang, *Complex Analysis*, Springer, New York, 2003.
2. R. Walter, *Real and Complex Analysis*, McGraw- Hill Book Co., New Delhi, 1986.
3. S. Ponnusamy, *Foundations of Complex Analysis*, Narosa Publication House, New Delhi, 1995.

**Course Title:** Discrete Mathematics

**Course Code:** MAT.558

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objectives:**

The objective of this course is to acquaint the students with the concepts in Discrete Mathematics. It includes the topics like logics, graph theory, trees and Boolean algebra.

**Unit-I**

**(14 Lecture Hours)**

**Mathematical Reasoning;** Basic logical operations, Conditional and bi-conditional statements, tautologies, Contradiction, Quantifiers, Propositional calculus. Recursively defined sequences. Solving recurrence relations: Generating functions. Basics of counting and the Pigeon-hole Principle.

**Unit-II**

**(15 Lecture Hours)**

**Set Theory:** Paradox in set theory, Inductive definition of sets and proof by induction; Peano postulates; **Relations:** Representation of relations by graphs, Properties of relations, Equivalence relations and partitions, Partial orderings, Linear and well-ordered sets;

**Unit-III**

**(16 Lecture Hours)**

**Graphs and Planar Graphs:** basic terminology, Special types of graphs. The handshaking theorem, Paths and circuits shortest paths. Connectivity of graphs. Isomorphism of graphs. Homeomorphic graphs. Eulerian and Hamiltonian graphs. Planar and non-planar graphs. Euler's formula. Graph coloring.

**Unit-IV**

**(15 Lecture Hours)**

**Trees:** Basic terminology. Binary trees. Tree traversing: Preorder, Postorder and inorder traversals. Minimum spanning trees, Prim's and Kruskal's algorithm. Boolean algebras: Boolean functions, Logic gates, Lattices and algebraic structures.

**Recommended books:**

1. K. H. Rosen, *Discrete Mathematics and its Applications*, McGraw Hill, Delhi, 2007.
2. K. D. Joshi, *Foundation of Discrete Mathematics*, J. Wiley & Sons, Delhi, 1989.

**Suggested Readings:**

1. D. S. Malik, and M. K. Sen, *Discrete Mathematical Structures Theory and Applications*, Thomson/Course Technology, 2004.
2. C. L. Liu, *Elements of Discrete Mathematics*, McGraw Hill, Delhi, 1986.

**Course Title:** Number Theory

**Course Code:** MAT.559

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objective:**

The objective of this course is to teach the fundamentals of different branches of Number Theory, namely, Geometry of Numbers and Analytic Number Theory.

**Unit-I**

**(15 Lecture Hours)**

Divisibility of Integers, Greatest common divisor, Euclidean algorithm. The fundamental theorem of arithmetic, Congruences, Residue classes and reduced residue classes.

**Unit-II**

**(15 Lecture Hours)**

Indices and its applications, Quadratic residues, Euler's criterion, Product of quadratic residues and quadratic non-residues, The Legendre symbol and its properties, Gauss's lemma, Quadratic reciprocity law, Jacobi symbol and its properties.

**Unit-III**

**(15 Lecture Hours)**

Chinese remainder theorem, Fermat's little theorem, Wilson's theorem, Euler's theorem. Arithmetic functions  $\sigma(n)$ ,  $d(n)$ ,  $\tau(n)$ ,  $\mu(n)$ , Order of an integer modulo  $n$ , primitive roots for primes, composite numbers having primitive roots.

**Unit-IV**

**(15 Lecture Hours)**

Representation of an integer as a sum of two and four squares. Diophantine equations  $ax + by = c$ ,  $x^2 + y^2 = z^2$  and its application to  $x^4 + y^4 = z^4$ . Farey sequences, Continued fractions.

**Recommended books:**

1. David, M. Burton, *Elementary Number Theory*, Tata McGraw-Hill, 7th Edition, New Delhi, 2012.
2. I. Niven, S. Zuckerman, and H. L. Montgomery, *Introduction to Number Theory*, Wiley Eastern, 1991.

**Suggested Readings:**

1. T. M. Apostol, *Introduction to Analytic Number Theory*, Springer Verlag, 1976.
2. G. H. Hardy and E. M. Wright, *An Introduction to the Theory of Number*, Oxford Univ. Press, U.K., 2008.
3. W. W. Adams and L. J. Goldstein, *Introduction to Number Theory*, Prentice Hall Inc., 1976.



**Course Title:** Operations Research

**Course Code:** MAT.560

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objective:**

The objective of this course is to acquaint the students with the concept of convex sets, their properties, Linear and nonlinear programming problems. The results, methods and techniques contained in this paper are very well suited to the realistic problems in almost every area

**Unit-I**

**(15 Lecture Hours)**

Operations Research and its Scope, Necessity of Operations Research in industry Mathematical formulation of linear programming problem, Linear Programming and examples, Convex Sets, Hyper plane, Open and Closed half-spaces, Feasible, Basic Feasible and Optimal Solutions, Extreme Point & graphical methods. Simplex method, Big-M method, Two phase method, Determination of Optimal solutions, Unrestricted variables.

**Unit-II**

**(15 Lecture Hours)**

Duality theory, Dual linear Programming Problems, Fundamental properties of dual problems, Complementary slackness, Unbounded solution in Primal. Dual Simplex Algorithm, Sensitivity analysis.

**Unit-III**

**(15 Lecture Hours)**

The General transportation problem, Duality in transportation problem, Loops in transportation tables, Solution of transportation problem, Test for optimality, Degeneracy, Transportation algorithm (MODI method), Minimization transportation problem. Assignment Problems: Mathematical formulation of assignment problem, Hungarian method for solving assignment problem, Traveling salesman problem and Sequencing.

**Unit -IV**

**(15 Lecture Hours)**

Replacement problem, replacement of items that Deteriorate, replacement of items that fails completely. Job Sequencing Problems; Introduction and assumption, Processing of n jobs through two machines, Processing of n jobs through three machines and m machines, Processing two jobs through n machines.

**Recommended books:**

1. H. A. Taha, *Operations Research - An Introduction*, Macmillan Publishing Company Inc., New York, 2006.
2. K. Swarup, P. K. Gupta, and M. Mohan, *Operations Research*, Sultan Chand & Sons, New Delhi, 2001.

**Suggested Readings:**

1. S. M. Sinha, *Mathematical Programming, Theory and Methods*, Delhi: Elsevier, 2006.
2. N. S. Kambo, *Mathematical Programming Techniques*, Affiliated East- West Press Pvt. Ltd., 1984, Revised Edition, New Delhi, 2005.
3. G. Hadley, *Linear Programming*, Narosa Publishing House, New Delhi, 1987.

**Semester-IV**

**Course Title:** Functional Analysis

**Course Code:** MAT.571

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objective:** The objective of this course is to introduce basic concepts, methods of Functional Analysis and its Applications. It is a first level course in Functional Analysis.

**Unit-I** **(14 Lecture Hours)**

**Fundamentals of Normed Linear Spaces:** Normed Linear spaces, Banach spaces and examples, finite dimensional normed spaces and subspaces, compactness and finite dimension. Quotient space of normed linear spaces and its completeness.

**Unit-II** **(15 Lecture Hours)**

Weak convergence and bounded linear transformations, Normed linear spaces of bounded linear transformations, Dual spaces with examples.

**Three Main Theorems on Banach Space:** Uniform boundedness theorem and some of its consequences, Open mapping and closed graph theorems.

**Unit-III** **(15 Lecture Hours)**

Hahn-Banach theorem for real linear spaces and its consequences, Complex linear spaces and normed linear spaces, Reflexive spaces, Solvability of linear equations in Banach spaces.

**Unit-IV** **(16 Lecture Hours)**

Inner product spaces. Hilbert spaces, Orthonormal sets, Bessel's inequality, Complete orthonormal sets and Parseval's Identity, Structure of Hilbert Spaces, Projection theorem, Riesz representation theorem, Adjoint of an operator on a Hilbert space, Reflexivity of Hilbert Spaces, Self-adjoint operators, normal and Unitary operators.

**Recommended books:**

1. B. V. Limaye, *Functional Analysis*, New Age International (P) Ltd, New Delhi, 1996.
2. E. Kreyszig, *Introductory Functional Analysis with Application*, Willey, 2007.

**Suggested Readings:**

1. S. K. Berberian, *Introduction to Hilbert Spaces*, AMS Chelsea Publishing, Rhode Island, 1996.
2. C. Goffman, and G. Pedrick, *First Course in Functional Analysis*, Prentice Hall of India, New Delhi, 1983.
3. F. K. Riesz, and B. S. Nagy, *Functional Analysis*, Dover Publications, 1990.
4. A. H. Siddiqui, *Functional Analysis*, Tata-McGraw Hill, New Delhi, 1987.

# Centre for Mathematics and Statistics, CUPB

33

**Course Title:** Project Work

**Course Code:** MAT.599

**Total Hours:** 120

L	T	P	Credits	Marks
0	0	16	8	200

**Course Title:** Riemannian Geometry

**Course Code:** MAT.572

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objective:** The objective of the course is to introduce students to the basic concepts of Riemannian geometry to prepare them for further studies and research in Riemannian geometry, Finsler geometry, Mathematical Physics and their applications in allied areas.

**Unit-I**

**(16 Lecture Hours)**

Differentiable manifolds, Examples: Spheres (in higher dimensions), Product of manifolds (construction of cylinder, Torus, n-torus as a product of manifolds). Smooth maps on a manifold, smooth map between two manifolds. Tangent space, Tangent bundle, Vector fields and Lie bracket. Covariant differentiation of vector fields and affine connection. Riemannian metric, Riemannian manifolds, Fundamental Theorem of Riemannian Geometry via Koszul Formula.

**Unit-II**

**(14 Lecture Hours)**

Tensors and Tensor fields (Riemannian metric as the most significant example), Tensorial property, Covariant differentiation of tensor fields, Riemann curvature tensor, Ricci tensor, Definition of sectional, Ricci and scalar curvatures, Isometries, notion of covering spaces, pull-back metrics via diffeomorphisms.

**Unit-III**

**(16 Lecture Hours)**

Covariant differentiation of a vector field along a curve with specific examples, Arc length and energy of a piecewise smooth curve, Geodesics as length minimizing curves, First variation of arc length, To show that geodesics are critical points of the fixed end point first variation formula, Exponential map, Geodesic completeness, Geodesic normal coordinates, Hopf-Renow Theorem (statement only), Geodesic variations, Jacobi fields and Gauss lemma.

**Unit-IV**

**(14 Lecture Hours)**

Second variation formula, the index form (Jacobi fields as minimizers of the Index form), Global differential geometry, spaces of constant sectional curvature, Bonnet-Myers, Cartan-Hadamard Theorem, Cartan's Theorems (on determination of metric by curvature).

**Recommended Books:**

1. J. M. Lee, *Riemannian Manifolds: An Introduction to Curvature*, GTM, Springer, 1<sup>st</sup> Edition, 1997.
2. B. O' Neill, *Semi-Riemannian geometry with Applications to Relativity*, Academic Press, New York, 1983.
3. M. P. Docarmo, *Riemannian geometry*, Birkhauser Boston, 1992.

**Suggested Readings:**

1. Marcel Berger, *A Panoramic View of Riemannian Geometry*, Springer; 1<sup>st</sup> Edition, 2003. Corr. 2<sup>nd</sup> printing 2007.
2. S. S. Chern, W. H. Chen and K. S. Lam, *Lectures on Differential Geometry*, World Scientific Publishing Co. Pte. Ltd., 2000.
3. W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian geometry*, 2<sup>nd</sup> Edition, Academic Press, New York, 2003.

# Centre for Mathematics and Statistics, CUPB

35

**Course Title:** Fluid Mechanics

**Course Code:** MAT.573

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

## Objective:

The objective of this course is to introduce to the fundamentals of the study of fluid motion and to the analytical approach to the study of fluid mechanics problems.

## Unit-I

(15 Lecture Hours)

Real fluids and ideal fluids, velocity of fluid at a point, streamlines, path lines, streak lines, velocity potential, vorticity vector, local and particle rate of change, equation of continuity, irrotational and rotational motion, acceleration of fluid, conditions at rigid boundary.

## Unit-II

(15 Lecture Hours)

Euler's equation of motion, Bernoulli's equation, their applications, Potential theorems, Axially symmetric flows, impulsive motion, Kelvin's theorem of circulation, equation of vorticity.

## Unit-III

(15 Lecture Hours)

Some Three Dimensional Flows: sources, sinks and doublets, images in rigid planes, images in solid sphere, Stoke's stream function.

## Unit-IV

(15 Lecture Hours)

Two Dimensional Flows: complex velocity potential, Milne Thomson circle theorem and applications, theorem of Blasius, vortex rows, Karman Vortex Street.

## Recommended books:

1. F. Charlton, *Text Book of Fluid Dynamics*, CBS Publishers, Indian Edition, Delhi, 2004
2. L. D. Landau, and E. M. Lipschitz, *Fluid Mechanics*, Pergamon Press Ltd., London, 1987.
3. G. K. Batchelor, *An Introduction to Fluid Mechanics*, Cambridge University Press, New York, 1967.

## Suggested Reading:

1. P. K. Kundu, and I. M. Cohen. *Fluid Mechanics*, Hardcover (India) Pvt.Ltd., Delhi, 2003.
2. G.K. Batechelor, *An Introduction to Fluid Dynamics*, Cambridge Press, 2002
3. H. Schliting and K. Gersten, *Boundary Layer Theory*, Springer, 8<sup>th</sup> Edition, 2004
4. L. Rosenhead, *Laminar Boundary Layers*, Dover Publications, 1963
5. P.G. Drazin, and W. H. Reid, *Hydrodynamic Stability*, Cambridge Press, 2004

**Course Title:** Advanced Numerical Analysis

**Course Code:** MAT.574

L	T	P	Credits	Marks
4	0	0	4	100

**Total Hours:** 60

**Objectives:** The objective of the course is to familiarize the students about some advanced numerical techniques e.g. solving systems of nonlinear equations, linear system of equations, Eigen value problems, Interpolation and Approximation techniques and their use in differentiation and integration, differential equations etc.

**UNIT- I**

**(16 Lecture Hours)**

**Non-Linear Equations:** Methods for multiple roots, Muller's, Iteration and Newton-Raphson method for non-linear system of equations, and Newton-Raphson method for complex roots.

**Polynomial Equations:** Descartes' rule of signs, Birge-Vieta, Bairstow and Giraffe's methods.

**System of Linear Equations:** Triangularization, Cholesky and Partition methods, SOR method with optimal relaxation parameters.

**UNIT-II**

**(14 Lecture Hours)**

**Eigen-Values of Real Symmetric Matrix:** Similarity transformations, Gerschgorin's bound(s) on eigenvalues, Jacobi, Givens, Householder and Rutishauser methods.

**Interpolation and Approximation:** B - Spline and bivariate interpolation, Gram-Schmidt orthogonalisation process and approximation by orthogonal polynomial, Legendre and Chebyshev polynomials and approximation.

**UNIT- III**

**(14 Lecture Hours)**

**Differentiation and Integration:** Differentiation and integration using cubic splines, Romberg integration and multiple integrals.

**Ordinary Differential Equations:** Shooting and finite difference methods for second order boundary value problems, Applications of cubic spline to ordinary differential equation of boundary value type.

**UNIT- IV**

**(16 Lecture Hours)**

**Partial Differential Equations:** Finite difference methods for Elliptic, Parabolic and Hyperbolic partial differential equations.

**Recommended Books:**

1. C. F. Gerald and P. O. Wheatly, *Applied Numerical Analysis*, 7<sup>th</sup> Edition, Pearson LPE, 2009.
2. R. S. Gupta, *Elements of Numerical Analysis*, Cambridge University Press, 2<sup>nd</sup> Edition, 2015.
3. R.L. Burden and J. D. Faires, *Numerical Analysis*, 9<sup>th</sup> Edition, Cengage Learning, 2011.

**Suggested Readings:**

1. K. Atkinson, *An Introduction to Numerical Analysis*, John Wiley & Sons, 2<sup>nd</sup> Edition, 1989.
2. M. K. Jain, S.R.K. Iyengar and R.K. Jain, *Numerical Methods for Scientific and Engineering Computation*, 6<sup>th</sup> Edition, New Age International, New Delhi, 2015.
3. S.D. Conte and Carl D. Boor, *Elementary Numerical Analysis: An Algorithmic Approach*, Tata McGraw Hill, 2005.

**Course Title:** Algebraic Topology

**Course Code:** MAT.575

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objective:** The objective of this course is to introduce the student's concept in Algebraic topology so that they can pursue research in this field and its allied areas.

**Unit-I**

**(14 Lecture Hours)**

The Fundamental group: Homotopy of paths, Homotopy classes, The Fundamental group, Change of base point, Topological invariance, Covering spaces, The Fundamental group of the circle.

**Unit-II**

**(14 Lecture Hours)**

Retractions and fixed points, No Retraction Theorem, The Fundamental theorem of Algebra, The Borsuk-Ulam theorem, The Bisection theorem, Deformation Retracts and Homotopy type, Homotopy invariance.

**Unit-III**

**(16 Lecture Hours)**

Direct sums of Abelian Groups, Free products of groups, Uniqueness of free products, Least normal subgroup, Free groups, Generators and relations, The Seifert-Van Kampen theorem, The Fundamental group of a wedge of circles.

**Unit-IV**

**(16 Lecture Hours)**

Classification of covering spaces: Equivalence of covering spaces, The general lifting lemma, The universal covering space, Covering transformation, Existence of covering spaces.

**Recommended Books:**

1. James R. Munkres, *Elements of Algebraic Topology*, Perseus Books, 1995.
2. A. Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.
3. Satya Deo, *Algebraic Topology: A Primer (Texts and Readings in Mathematics)*, Hindustan Book Agency, 2003.

**Suggested Readings:**

1. M. A. Armstrong, *Basic Topology*, UTM Springer, 2000.
2. E. H. Spanier, *Algebraic Topology (2nd edition)*, Springer-Verlag, New York, 2000.
3. J. J. Rotman, *An Introduction to Algebraic Topology, Text in Mathematics, No. 119*, Springer, New York, 2004.
4. W. S. Massey, *A Basic Course in Algebraic Topology*, SPRINGER (SIE), 2007.
5. M. J. Greenberg and J. R. Harper, *Algebraic Topology: A First Course*, 2<sup>nd</sup> Edition, Addison-Wesley Publishing Co, 1997.

**Course Title:** Lie Groups and Lie Algebra

**Course Code:** MAT.576

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objective:**

The aim of this course is to make the students learn basic concepts of Lie groups and Lie algebra, so as to enable the students to understand further topics related to solution of differential equations.

**Unit I**

**(15 Lecture Hours)**

Differential Manifolds Topological manifolds, Charts, Atlases and smooth structure, Smooth maps and diffeomorphism, Partitions of Unity, Tangent space, Tangent map, Vector fields and 1-forms.

**Unit II**

**(15 Lecture Hours)**

Lie Groups Definition and examples, Linear Lie groups, Lie group homomorphism, Lie algebra and the exponential map, Adjoint representation, Homogeneous spaces, Baker-Campbell-Housdorff formula.

**Unit III**

**(15 Lecture Hours)**

Lie Algebras Definition and examples, Classical Lie algebras, Solvable and nilpotent Lie algebras, Lie and Engel theorems, Semi-simple and reductive algebras, Semi-simplicity of Classical Lie algebras

**Unit IV**

**(15 Lecture Hours)**

Semisimple Lie algebras; Killing form; Jordan decomposition; Engel's Theorem, Cartan subalgebra and Root space decomposition, Geometry of Root systems, Simple roots and Weyl group, Classification of root systems; Examples

**Recommended Books:**

1. J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Graduate Text in Mathematics, 9, Springer-Verlag, 1980.
2. N. Jacobson, *Lie Algebras*, Wiley-Interscience, New York, 1962.
3. J. P. Serre, *Lie Algebras and Lie Groups*, Benjamin, New York, 1965.
4. N. Bourbaki, *Lie Groups and Lie Algebras*, Springer Science & Business Media, 1998.

**Suggested Readings:**

1. K. J. Alexander, *An Introduction to Lie Groups and Lie Algebras*, Cambridge University Press.
2. S. Kumaresan, *Differential Geometry and Lie Groups*, Hindustan Book Agency.
3. B. Hall, *Lie Groups, Lie Algebras, and Representations: An Elementary Introduction*, Second Edition, Springer.
4. P. J. Olver, *Application of Lie Groups to Differential Equations*, Second Edition, Springer.



**Course Title:** Finite Element Analysis

**Course Code:** MAT.577

**Total Hours:** 60

L	T	P	Credits	Marks
4	0	0	4	100

**Objective:**

The aim of this course is to make the students learn fundamental concepts of finite elements so as to enable the students to understand further topics related to solution of differential equations. Finite element analysis is a helpful tool to solve a variety of problems of science and engineering related to fluid flows, structures etc.

**Unit-I**

**(14 Lecture Hours )**

General theory of finite element methods, Difference between finite element and finite difference, Review of some integral formulae, Concept of discretization, Convergence requirements, Different coordinates, One dimensional finite elements, shape functions, stiffness matrix, connectivity, boundary conditions, equilibrium equation, FEM procedure.

**Unit-II**

**(16 Lecture Hours)**

Generalization of the finite element concepts-weighted residual and variational Approaches (Ritz method, Galerkin method, collocation method etc.) Numerical integration, Interpolation formulas and shape functions, Axis symmetric formulations, solving one-dimensional problems.

**Unit-III**

**(14 Lecture Hours)**

Two dimensional finite element methods, Element types: triangular, rectangular, quadrilateral, sector, curved, isoperimetric elements and numerical integration, two dimensional boundary value problems, connectivity and nodal coordinates, variational functions, triangular elements and area coordinates, transformations, cylindrical coordinates.

**Unit-IV**

**(16 Lecture Hours)**

Three dimensional finite elements, higher order finite elements, element continuity, plate finite elements, Application of finite element methods to elasticity problems and heat transfer problems.

**Recommended Books:**

1. C. S. Desai, *Introductory Finite Element Method*, Boca Raton: CRC Press, 2001.
2. G. D. Smith, *Numerical Solution of Partial Differential Equations*, Oxford: Clarendon Press, 1986.

**Suggested Readings:**

1. B. B. Bradie, *A Friendly Introduction to Numerical Analysis*, Pearson New Delhi, 2005.
2. J. N. Reddy, *An Introduction to Finite Element Methods*, Delhi: McGraw-Hill Higher Education, 2005.
3. D. Braess, Schumaker and Larry L. *Finite Elements: Theory, Fast Solvers, and Applications in Solid Mechanics*, New York: Cambridge University Press, 2001.