

Central University of Punjab, Bathinda



Course Scheme & Syllabus for Ph. D. Course Work in Mathematics / Statistics

Course structure for Ph.D. in Mathematics / Statistics

Students can move into the Ph.D. programme after successful completion of one semester course work, provided they meet the requirements specified by the university.

Structure for course work for PhD in Mathematics / Statistics

	Semester I						
	S. No.	Subject Code	Subject Name	Credit Hours			Maximum Marks
				Lecture	Practical	Credits	
Compulsory	1.	PHDMS.701	Research Methodology and Computer Applications	4		4	100
	2.	PHDMS.702	Review Writing and Seminar		8	4	100
Opt any two out of the following elective courses offered							
	3.	PHDMS.703	Real Analysis	4		4	100
	4.	PHDMS.704	Linear Algebra	4		4	100
	5.	PHDMS.705	Symmetries and Differential Equations	4		4	100
	6.	PHDMS.706	Fractional Calculus	4		4	100
	7.	PHDMS.707	Advanced Partial Differential Equations	4		4	100
	8.	PHDMS.708	Advanced Numerical Analysis	4		4	100
	9.	PHDMS.709	Operations Research	4		4	100
	10.	PHDMS.710	Number Theory	4		4	100
	11.	PHDMS.711	Advanced Algebra	4		4	100
	12.	PHDMS.712	Functional Analysis	4		4	100
	13.	PHDMS.713	Advanced Complex Analysis	4		4	100
	14.	PHDMS.714	Differential Equations and Boundary-Value Problems	4		4	100
	15.	PHDMS.715	Potential Flow of Fluids and Water-wave Theory	4		4	100

16.	PHDMS.716	Topology	4		4	100
17.	PHDMS.717	Differential Geometry of Curves and Surfaces	4		4	100
18.	PHDMS.718	Differential Topology	4		4	100
19.	PHDMS.719	Algebraic Topology	4		4	100
20.	PHDMS.720	Riemannian Geometry	4		4	100
21.	PHDMS.721	Riemann Finsler Geometry	4		4	100
22.	PHDMS.722	Lie Groups and Lie Algebra	4		4	100
23.	PHDMS.723	Probability Theory	4		4	100
24.	PHDMS.724	Stochastic Processes and Queuing Theory	4		4	100
25.	PHDMS.725	Reliability Theory	4		4	100
26.	PHDMS.726	Sampling Theory	4		4	100
Total					16	400

**Syllabi for Ph. D. Course work
Semester I**

Course Title: Research Methodology and Computer Applications

Course Code: PHDMS.701

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Course Objective: The objective of this course is to ensure that a student learns basis of scientific research. Also the student will develop understanding of computer hardware systems and its few basic applications.

Unit-I (15 Lecture Hours)

Introduction: Meaning, Objectives, Characteristics, Significance, and Types of Research; Research Approaches, Research Methods vs. Research Methodology, Research Process, and Criteria of Good Research.

Literature Survey and Review: Meaning of Literature Survey and Review, Sources of Literature, Methods of Literature Review, and Techniques of Writing the Reviewed Literature.

Formulating Research Problem: Understanding a Research Problem, Selecting the Research Problem, Steps in Formulation of a Research Problem, Formulation of Research Objectives, and Construction of Hypothesis.

Unit-II (15 Lecture Hours)

Research Design: Meaning of and Need for Research Design, Characteristics of a Good Research Design, Different Research Designs, Basic Principles of Experimental Designs, Data Collection, Processing, and Interpretation.

Report Writing: Types of Reports – Technical and Popular Reports, Significance of Report Writing, Different Steps in Writing Report, Art of Writing Research Proposals, Research Papers, Project Reports, and Dissertations/Thesis; Basics of Citation and Bibliography/Reference Preparation Styles; Report Presentation: Oral and Poster Presentations of Research Reports.

Unit III (15 Lecture Hours)

Fundamentals of Computers and Internet: Block Diagram of computer, Hardware components, Introduction to computer network and world wide web, Sharing data over network, Internet terminology, Searching over internet, Google: advance search operations, Email, Checking Plagiarism using internet

Unit IV (15 Lecture Hours)

Introduction to Microsoft Office: Creating and saving documents, Text formatting, Tables, Document review option, Mail merge, Inserting table of contents, Reference management. Introduction to spreadsheet and Microsoft excel, Text formatting, Formulas, Charts, Table formatting, Sorting records, Filtering the content. Introduction to Microsoft power point, Layout selection, Designing and formatting slides, Slide design and background formatting,

Recommended Books:

1. Kothari, C.R. and G. Garg (2014): *Research Methodology: Methods and Techniques*, 3rd ed., New Age International Pvt. Ltd. Publisher
2. Kumar, R. (2014): *Research Methodology – A Step-By-Step Guide for Beginners*, 4th ed., Sage Publications
3. D. Gookin, *MS Word for Dummies*. Wiley, 2007.
4. G. Harvey, *MS Excel for Dummies*. Wiley, 2007.
5. P. K. Sinha, *Computer Fundamentals*, BPB Publications, 2006.
6. J. Anderson, *Thesis and Assignment Writing*, 4th ed., Wiley, USA, 2001.
7. Catherine Dawson, *Practical Research Methods*, New Delhi, UBS Publishers' Distributors, 2014.
8. David E. Gray, *Doing Research in the Real World*. London, UK: Sage Publications, 2004.

Syllabi applicable for Admissions in Ph. D. (Mathematics/Statistics), 2017

Course Title: Review Writing and Presentation

Course Code: PHDMS.702

L	T	P	Credits	Marks
0	0	8	4	100

Total Hours: 120

Objective: The objective of this course would be to ensure that the student learns the aspects of the Review writing and seminar presentation. Herein the student shall have to write a review of existing scientific literature with simultaneous identification of knowledge gaps that can be addressed through future work.

The evaluation criteria for “Review Writing and Presentation” shall be as follows:

Maximum Marks: 100

S. No.	Criteria	Marks
1.	Review of literature	25
2.	Identification of gaps in knowledge	15
3.	References	10
4.	Content of presentation	15
5.	Presentation Skills	20
6.	Handling of queries	15
Total		100

Course Title: Real Analysis

Course Code: PHDMS.703

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Objective: The aim of this course is to make the students learn fundamental concepts of metric spaces, the Riemann-Stieltjes integral as a generalization of Riemann Integral, Sequence and series of functions, the calculus of several variables and some basic theorems.

Unit-I **(15 Lecture Hours)**

Set Theory: Finite, countable and uncountable sets

Metric spaces: Definition and examples, Open and closed sets, Compact sets, Elementary properties of compact sets, k - cells, Compactness of k -cells, Compact subsets of Euclidean space \mathbb{R}^k , Perfect sets, Cantor set, Separated sets, Connected sets in a metric space, Connected subsets of real line.

Unit-II **(15 Lecture Hours)**

Sequences in Metric spaces: Convergent sequences, Subsequences, Cauchy sequences, Complete metric space, Cantor's intersection theorem, Category of a set and Baire's category theorem. Examples of complete metric space, Banach contraction principle.

Unit-III **(15 Lecture Hours)**

Continuity: Limits of functions (in Metric spaces), Continuous functions, Continuity and compactness, Continuity and connectedness, Discontinuities, Monotonic functions, Uniform continuity.

Riemann Stieltje's Integral: Definition and existence of Riemann Stieltje's integral, Properties of integral. Integration and Differentiation. Fundamental Theorem of Calculus, 1st and 2nd Mean Value Theorems of Riemann Stieltje's integral.

Unit-IV **(15 Lecture Hours)**

Sequences and series of functions: Problem of interchange of limit processes for sequences of functions, Uniform convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation, equicontinuous families of functions, Stone Weierstrass Theorem.

Recommended Books:

1. W. Rudin, *Principles of Mathematical Analysis, 3rd edition*, McGraw Hill, Kogakusha, International student edition, 1976.
2. H. L. Royden, *Real Analysis, 3rd edition*, Macmillan, New York & London 1988.
3. S. C. Malik, *Mathematical Analysis*, Wiley Eastern Ltd., 2010.
4. E. C. Titchmarsh, *The Theory of functions, 2nd Edition*, U.K. Oxford University Press 1961.
5. T. M. Apostol, *Mathematical Analysis*, Addition –Wesley, 2002.
6. G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw-Hill Ltd, 2003.
7. A. Kumar and S. Kumaresan, *A Basic Course in Real Analysis*, Narosa, Publishing House, 27 January 2014.
8. R. G. Bartle, *The Elements of Real Analysis*, John Willey and Sons, 1976.

Course Title: Linear Algebra

Course Code: PHDMS.704

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Objective:

The concepts and techniques from linear algebra are of fundamental importance in many scientific disciplines. The main objective is to introduce basic notions in linear algebra that are often used in mathematics and other sciences. The emphasis will be to combine the abstract concepts with examples in order to intensify the understanding of the subject.

Unit I

(14 Lecture Hours)

Vector Space: Vector spaces, Subspaces, Direct sum of subspaces, Linear dependence and independence, Basis and dimensions, Linear transformations, Algebra of linear transformations, Dual spaces, Matrix representation of a linear transformation, Rank and nullity of a linear transformation, Invariant subspaces. Change of basis.

Unit I

(15 Lecture Hours)

Characteristic polynomial and minimal polynomial of a linear transformation, Cayley Hamilton theorem, Eigenvalues and eigenvectors of a linear transformation, Diagonalization and triangularization of a matrix, Characteristic polynomial and minimal polynomial of block matrices. Canonical forms, Diagonal forms, Triangular forms, Jordan canonical forms, rational canonical forms, Quotient spaces.

Unit III

(14 Lecture Hours)

Linear functional, Dual space, Dual basis, Annihilators, Bilinear forms, Symmetric bilinear forms, Sylvester's theorem, quadratic forms, Hermitian forms. Reduction and classification of quadratic forms.

Unit IV

(13 Lecture Hours)

Inner product spaces. Norms and distances, Orthonormal basis, Orthogonality, Schwartz inequality, The Gram-Schmidt orthogonalization process. Orthogonal and positive definite matrices. The Adjoint of a linear operator on an inner product space, Normal and self-adjoint operators, Unitary and orthogonal operators.

Recommended books:

1. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, *First Course in Linear Algebra*, Wiley Eastern, Delhi, 2003.
2. J. Gilbert and L. Gilbert: *Linear Algebra and Matrix Theory*, Cengage Learning, 2004.
3. I. N. Herstein, *Topics in Algebra* 2nd Edition, Wiley Eastern Limited, New Delhi, 2006.
4. V. Bist and V. Sahai, *Linear Algebra*, Narosa, Delhi, 2002.
5. K. Hoffman and R. Kunze: *Linear Algebra* 2nd Edition, Pearson Education (Asia) Pvt. Ltd/ Prentice Hall of India, 2004.

Course Title: Symmetries and Differential Equations

Course Code: PHDMS.705

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Unit I

(15 Lecture Hours)

Dimensional Analysis: Buckingham Pi-theorem, Assumptions behind dimensional analysis, Conclusions from dimensional analysis, Proof of the Buckingham Pi-theorem and examples, Application of dimensional analysis to partial differential equations, Generalization of dimensional analysis, Invariance of partial differential equations under scaling of variables

Unit II

(15 Lecture Hours)

Lie Group of Transformations: Groups, Examples of groups, Groups of transformations, One-parameter Lie group of transformations, Examples of one-parameter Lie groups of transformations, Infinitesimal transformations: First fundamental theorem of Lie, Infinitesimal generators, Invariant functions

Unit III

(15 Lecture Hours)

Canonical coordinates, Invariant surfaces, Invariant curves, Invariant points, Extended transformations: Extended group transformations-one dependent and one independent variable, Extended infinitesimal transformations-one dependent and one independent variable, Extended transformations-one dependent and n independent variables

Unit IV

(15 Lecture Hours)

Multi-parameter Lie groups of transformations; Lie algebras, r-parameter Lie groups of transformations, Lie algebras, Examples of Lie algebras, Solvable Lie algebras

Recommended Books:

1. G. W. Bluman and A. C. Anco, *Symmetry and Integration Methods for Differential Equations*, Appl. Math. Sci., 154, Springer, New York, 2002.
2. G. W. Bluman and S. Kumei, *Symmetries and Differential Equations*, Appl. Math.Sci., Springer-Berlin, 1989.
3. P. J. Olver, *Applications of Lie Groups to Differential Equations*, Springer-Verlag, New York 1993.
4. L. V. Ovsianikov, *Group Properties of Differential Equations*, Novosibirsk, Moscow, 1962.

Course Title: Fractional Calculus

Course Code: PHDMS.706

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Unit I

(16 Lecture Hours)

Special Functions of Fractional Calculus: Gamma function, Some properties of Gamma function, Beta function, Contour integral representation. Fractional derivatives and integrals, GrunwaldLetnikov Fractional derivatives, Riemann-Liouville fractional derivatives, Caputo's fractional derivative, The Leibniz rule for fractional derivatives, Geometric and physical interpretation of fractional integration and fractional differentiation.

Unit II

(14 Lecture Hours)

Sequential fractional derivatives. Left and right fractional derivatives. Properties of fractional derivatives. Laplace transforms of fractional derivatives. Fourier transforms of fractional derivatives. Mellin transforms of fractional derivatives.

Unit III

(15 Lecture Hours)

Linear Fractional Differential Equations: Fractional differential equation of a general form. Existence and uniqueness theorem as a method of solution. Dependence of a solution on initial conditions. The Laplace transform method. Standard fractional differential equations. Sequential fractional differential equations.

Unit IV

(15 Lecture Hours)

Fractional Differential Equations: Introduction, Linearly independent solutions, Solutions of the homogeneous equations, Solution of the non-homogeneous fractional differential equations, Reduction of fractional differential equations to ordinary differential equations. Semi differential equations

Recommended Books:

1. K. B. Oldham & J. Spanier, *The Fractional Calculus: Theory and Applications of Differentiation and Integration to Arbitrary Order*, Dover Publications Inc, 2006.
2. K. S. Miller & B. Ross., *An Introduction to the Fractional Calculus and Fractional Differential Equations Hardcover*, Wiley Blackwell, 1993.
3. I. Podlubny, *Fractional Differential Equations*, Academic Press, 1998

Course Title: Advanced Partial Differential Equations

Course Code: PHDMS.707

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Objectives:

The objective of this course is to equip the students with knowledge of some advanced concepts related to partial differential equations and to understand some basic approaches to mathematical oriented PDEs.

Unit-I

(16 Lecture Hours)

Distribution: Test Functions and Distributions, Examples, Operations on Distributions, Supports and Singular Supports, Convolution, Fundamental Solutions, Fourier Transform, Schwartz space, Tempered Distributions.

Sobolev Spaces: Basic properties, Approximation by smooth functions, Extension theorems, Compactness theorems, Dual spaces, Functional order spaces, Trace spaces, Trace theory, Inclusion theorem.

Unit-II

(15 Lecture Hours)

Weak Solutions of Elliptic Boundary Value Problems: Variational problems, Weak formulation of Elliptic PDE, Regularity, Galerkin Method, Maximum principles, Eigenvalue problems, Introduction to finite element methods.

Unit-III

(14 Lecture Hours)

Evolution Equations: Unbounded linear operators, C_0 – Semigroups, Hille-Yosida theorem, Contraction Semigroup on Hilbert Spaces, Heat equation, Wave equation, Schrodinger equation, Inhomogeneous equations.

Unit-IV

(15 Lecture Hours)

Calculus of Variations: Euler-Lagrange Equation, Second variation, Existence of Minimizers (Coactivity, Lower Semi-continuity, Convexity), Regularity, Constraints (Nonlinear Eigenvalue problems, Variational Inequalities, Harmonic maps, Incompressibility), Critical points (Mountain Pass theorem and Applications to Elliptic PDE).

Recommended Books:

1. S. Kesavan, *Topics in Functional Analysis and Application*, Wiley-Eastern, New International, New Delhi, 1999.
2. L. C. Evans, *Partial Differential Equations. Graduate Studies in Mathematics*, American Mathematical Society, 2nd Edition, Indian Reprint, 2014.

Course Title: Advanced Numerical Analysis

L	T	P	Credits	Marks
4	0	0	4	100

Course Code: PHDMS.708

Total Hours: 60

Objectives: The objective of the course is to familiarize the students about some advanced numerical techniques e.g. solving systems of nonlinear equations, linear system of equations, Eigen value problems, Interpolation and Approximation techniques and their use in differentiation and integration, differential equations etc.

UNIT- I

(16 Lecture Hours)

Non-Linear Equations: Methods for multiple roots, Muller's, Iteration and Newton-Raphson method for non-linear system of equations, and Newton-Raphson method for complex roots.

Polynomial Equations: Descartes' rule of signs, Birge-Vieta, Bairstow and Giraffe's methods. System of Linear Equations: Triangularization, Cholesky and Partition methods, SOR method with optimal relaxation parameters.

UNIT-II

(14 Lecture Hours)

Eigen-Values of Real Symmetric Matrix: Similarity transformations, Gerschgorin's bound(s) on eigenvalues, Givens, Householder and Rutishauser methods.

Interpolation and Approximation: B - Spline and bivariate interpolation, Gram-Schmidt orthogonalisation process and approximation by orthogonal polynomial, Legendre and Chebyshev polynomials and approximation.

UNIT- III

(14 Lecture Hours)

Differentiation and Integration: Differentiation and integration using cubic splines, Romberg integration and multiple integrals.

Ordinary Differential Equations: Shooting and finite difference methods for second order boundary value problems, Applications of cubic spline to ordinary differential equation of boundary value type.

UNIT- IV

(16 Lecture Hours)

Partial Differential Equations: Finite difference methods for Elliptic, Parabolic and Hyperbolic partial differential equations.

Recommended Books:

1. K. Atkinson, *An Introduction to Numerical Analysis*, John Wiley & Sons, 2nd Edition, 1989.
2. R. L. Burden and J. D. Faires, *Numerical Analysis*, 9th Edition, Cengage Learning, 2011.
3. S. D. Conte, S.D. and Carl D. Boor, *Elementary Numerical Analysis: An Algorithmic Approach*, Tata McGraw Hill 2005.
4. C. F. Gerald and P. O. Wheatly, *Applied Numerical Analysis*, 7th Edition, Pearson LPE, 2009.
5. R. S. Gupta, *Elements of Numerical Analysis*, Cambridge University Press, 2nd Edition, 2015.
6. M. K. Jain, S.R.K. Iyengar and R.K. Jain, *Numerical Methods for Scientific and Engineering Computation*, 6th Edition, New Age International, New Delhi, 2015.

Course Title: Operations Research

Course Code: PHDMS.709

L	T	P	Credits	Marks
4	0	0	4	100

Total Hours: 60

Objective:

The objective of this course is to acquaint the students with the concept of convex sets, their properties, Linear and nonlinear programming problems. The results, methods and techniques contained in this paper are very well suited to the realistic problems in almost every area

Unit-I (15 Lecture Hours)

Operations Research and its Scope, Necessity of Operations Research in industry Mathematical formulation of linear programming problem, Linear Programming and examples, Convex Sets, Hyper plane, Open and Closed half-spaces, Feasible, Basic Feasible and Optimal Solutions, Extreme Point & graphical methods. Simplex method, Big-M method, Two phase method, Determination of Optimal solutions, Unrestricted variables.

Unit-II (15 Lecture Hours)

Duality theory, Dual linear Programming Problems, Fundamental properties of dual problems, Complementary slackness, Unbounded solution in Primal. Dual Simplex Algorithm, Sensitivity analysis.

Unit-III (15 Lecture Hours)

The General transportation problem, Duality in transportation problem, Loops in transportation tables, Solution of transportation problem, Test for optimality, Degeneracy, Transportation algorithm (MODI method), Minimization transportation problem. Assignment Problems: Mathematical formulation of assignment problem, Hungarian method for solving assignment problem, Traveling salesman problem and Sequencing.

Unit -IV (15 Lecture Hours)

Replacement problem, replacement of items that Deteriorate, replacement of items that fails completely. Job Sequencing Problems; Introduction and assumption, Processing of n jobs through two machines, Processing of n jobs through three machines and m machines, Processing two jobs through n machines.

Recommended books:

1. H. A. Taha, *Operations Research - An Introduction*, Macmillan Publishing Company Inc., New York, 2006.
2. K. Swarup, P. K. Gupta, and M. Mohan, *Operations Research*, Sultan Chand & Sons, New Delhi, 2001.
3. S. M. Sinha, *Mathematical Programming, Theory and Methods*, Delhi: Elsevier, 2006.
4. N. S. Kambo, *Mathematical Programming Techniques*, Affiliated East- West Press Pvt. Ltd., 1984, Revised Edition, New Delhi, 2005.
5. G. Hadley, *Linear Programming*, Narosa Publishing House, New Delhi, 1987.

Course Title: Number Theory

Course Code: PHDMS.710

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Objective:

The objective of this course is to teach the fundamentals of different branches of Number Theory, namely, Geometry of Numbers and Analytic Number Theory.

Unit I

(15 Lecture Hours)

Divisibility of Integers, Greatest common divisor, Euclidean algorithm. The fundamental theorem of arithmetic, Congruences, Residue classes and reduced residue classes.

Unit II

(15 Lecture Hours)

Indices and its applications, Quadratic residues, Euler's criterion, Product of quadratic residues and quadratic non-residues, The Legendre symbol and its properties, Gauss's lemma, Quadratic reciprocity law, Jacobi symbol and its properties.

Unit III

(15 Lecture Hours)

Chinese remainder theorem, Fermat's little theorem, Wilson's theorem, Euler's theorem. Arithmetic functions $\sigma(n)$, $d(n)$, $\tau(n)$, $\mu(n)$, Order of an integer modulo n , primitive roots for primes, composite numbers having primitive roots.

Unit IV

(15 Lecture Hours)

Representation of an integer as a sum of two and four squares. Diophantine equations $ax + by = c$, $x^2 + y^2 = z^2$ and its application to $x^4 + y^4 = z^4$. Farey sequences, Continued fractions.

Recommended books:

1. David, M. Burton, *Elementary Number Theory*, Tata McGraw-Hill, 7th Edition, New Delhi, 2012.
2. I. Niven, S. Zuckerman, and H. L. Montgomery, *Introduction to Number Theory*, Wiley Eastern, 1991.
3. T. N. Apostol, *Introduction to Analytic Number Theory*, Springer Verlag, 1976.
4. G. H. Hardy and E. M. Wright, *An Introduction to the Theory of Number*, Oxford Univ. Press, U.K., 2008.
5. W. W. Adams and L. J. Goldstein, *Introduction to Number Theory*, Prentice Hall Inc., 1976.

Course Title: Advanced Algebra

Course Code: PHDMS.711

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Objective:

This course is for students who wish to pursue research work in Algebra.

Unit I **(14 Lecture Hours)**

Groups: Groups: Jordan Holder theorem; solvable groups; symmetric and alternating groups; nilpotent groups; groups acting on sets; Sylow theorems; free groups.

Unit-II **(16 Lecture Hours)**

Rings and Modules: Noetherian and Artinian rings and modules; semi-simple rings; Hilbert basis theorem; Principal ideal domains and unique factorisation domains; modules over PID; linear algebra and Jordan canonical form; structure theorems for semi-simple rings. Representation theory of finite groups.

Unit-III **(14 Lecture Hours)**

Field Theory: Basic concepts of field theory, Steinitz theorem, Extension of fields, algebraic and transcendental extensions. Algebraically closed fields, Splitting fields, Separable and inseparable extensions, Normal extension, Multiple roots, Finite fields, Perfect fields.

Unit-IV **(16 Lecture Hours)**

Galois Theory: Automorphism groups, Fixed fields, Galois extensions, The fundamental theorem of Galois theory, Cyclotomic extensions, and Cyclic extensions, Applications of cyclotomic extensions and Galois theory to the constructability of regular polygons, Solvability of polynomials by radicals.

Recommended Books:

1. P. B. Bhattacharya, S.K. Jain and S.R. Nagpaul, *First Course in Linear Algebra*, Wiley Eastern, Delhi, 2008.
2. J. Gilbert and L. Gilbert, *Linear Algebra and Matrix Theory*, Academic Press, 2004.
3. I. N. Herstein, *Topics in Algebra*, 2nd Edition, Wiley Eastern Limited, New Delhi, 2006
4. V. Bist and V. Sahai, *Linear Algebra*, Narosa, Delhi, 2002.
5. J. P. Escofier, *Galois Theory*, Springer-Verlag, 2000.
6. I. Stewart, *Galois Theory*, Chapman and Hall, 2003.
7. B. Hartley and T. O. Hawkes, *Rings, Modules and Linear Algebra*, Chapman and Hall, 1970.
8. C. Musili, *Rings and Modules*, 2nd Revised Edition, Narosa Publishing House, New Delhi, 1994.
9. M. Artin, *Algebra*, 2nd Edition, Prentice Hall of India, 2011.
10. D. S. Dummit and R. M. Foote, *Abstract Algebra*, 3rd Edition, John Wiley, 2011.

Course Title: Functional Analysis

Course Code: PHDMS.712

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Objective: The objective of this course is to introduce basic concepts, methods of Functional Analysis and its Applications. It is a first level course in Functional Analysis.

Unit-I **(14 Lecture Hours)**

Fundamentals of Normed Linear Spaces: Normed Linear spaces, Banach spaces and examples, finite dimensional normed spaces and subspaces, compactness and finite dimension. Quotient space of normed linear spaces and its completeness.

Unit-II **(15 Lecture Hours)**

Weak convergence and bounded linear transformations, Normed linear spaces of bounded linear transformations, Dual spaces with examples.

Three Main Theorems on Banach Space: Uniform boundedness theorem and some of its consequences, Open mapping and closed graph theorems.

Unit-III **(15 Lecture Hours)**

Hahn-Banach theorem for real linear spaces and its consequences, Complex linear spaces and normed linear spaces, Reflexive spaces, Solvability of linear equations in Banach spaces.

Unit-IV **(16 Lecture Hours)**

Inner product spaces. Hilbert spaces, Orthonormal sets, Bessel's inequality, Complete orthonormal sets and Parseval's Identity, Structure of Hilbert Spaces, Projection theorem, Riesz representation theorem, Adjoint of an operator on a Hilbert space, Reflexivity of Hilbert Spaces, Self-adjoint operators, normal and Unitary operators.

Recommended books:

1. B. V. Limaye, *Functional Analysis*, New Age International (P) Ltd, New Delhi, 1996.
2. S. K. Berberian, *Introduction to Hilbert Spaces*, AMS Chelsea Publishing, Rhode Island, 1996.
3. C. Goffman, and G. Pedrick, *First Course in Functional Analysis*, Prentice Hall of India, New Delhi, 1983.
4. F. K. Riesz, and B. S. Nagy, *Functional Analysis*, Dover Publications, 1990.
5. A. H. Siddiqui, *Functional Analysis*, Tata-McGraw Hill, New Delhi, 1987.
6. E. Kreyszig, *Introductory Functional Analysis with Application*, Willey, 2007.

Course Title: Advanced Complex Analysis

Course Code: PHDMS.713

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Objectives:

This course is designed to enable the readers to understand further deeper topics of Complex Analysis and will provide basic topics needed for students to pursue research in pure Mathematics.

Unit-I

(16 Lecture Hours)

Harmonic function: Definition, Relation between a harmonic function and an analytic function, Examples, Harmonic Conjugate of a harmonic function, Poisson's Integral formula, Mean Value Property, The maximum & minimum principles for harmonic functions, Dirichlet Problem for a disc and uniqueness of its solution, Characterization of harmonic functions by mean value property.

Unit-II

(16 Lecture Hours)

Analytic continuation: Direct Analytic continuation, Analytic continuations along arcs, Homotopic curves, The Monodromy theorem, Analytic continuation via reflection. Harneck's principle. Open mapping theorem, normal families, The Riemann Mapping Theorem, Picard's theorem.

Unit-III

(14 Lecture Hours)

Weierstrass Elliptic functions: Periodic functions, Simply periodic functions, fundamental period, Jacobi's first and second question, Doubly periodic functions, Elliptic functions, Pair of Primitive Periods, Congruent points, First and Second Liouville's Theorem, Relation between zeros and poles of an elliptic function, Definition of Weierstrass elliptic function (z) and their properties, The differential equation satisfied by (z) [i.e., the relation between (z) and (z)], Integral formula for (z) , Addition theorem and Duplication formula for (z) .

Unit- IV

(13 Lecture Hours)

Weierstrass Zeta function: Weierstrass Zeta function and their properties, Quasi periodicity of (z) , Weierstrass sigma function (z) and their properties, Quasiperiodicity of (z) , Associated sigma functions.

Recommended Books:

1. J. B. Conway, *Functions of One Complex Variable*, Springer-Verlag International, USA, 1978.
2. L.V. Ahlfors, *Complex Analysis: An Introduction to the Theory of Analytic Functions of One Complex Variable*, McGraw-Hill Higher Education, New Delhi, 1979.
3. S. Lang, *Complex Analysis*, Springer, New York, 2003.
4. R. Walter, *Real and Complex Analysis*, McGraw- Hill Book Co., New Delhi, 1986.
5. S. Ponnusamy, *Foundations of Complex Analysis*, Narosa Publication House, New Delhi, 1995.

Course Title: Differential Equations and Boundary-Value Problems

Course Code: PHDMS.714

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Unit I

(16 Lecture Hours)

Existence and uniqueness of solutions of ODEs, Power series solution, Singular points, Some special functions. Nonlinear system of ODE : Preliminary concepts and definitions, The fundamental existence-uniqueness results, Dependence on initial conditions and parameters, The maximum interval of existence.

Unit II

(15 Lecture Hours)

Linearization, Stability and Liapunov functions, Saddle, Nodes, Foci and Centers, Normal form theory and Hamiltonian systems. Boundary value problems : Green's function method, Sturm-Liouville problem.

Unit III

(14 Lecture Hours)

First-order PDEs, Cauchy problem, Method of characteristics, Second-order PDEs, Classification, Characteristics and canonical forms. Elliptic boundary value problems : Maximum principle, Green's function,

Unit IV

(15 Lecture Hours)

Sobolev spaces, Variational formulations, Weak solutions, Lax-Milgram theorem, Trace theorem, Poincaré inequality, Energy estimates, Fredholm alternative, Regularity estimates, System of conservation laws, Entropy criteria.

Recommended Books:

1. L. Perko, *Differential Equations and Dynamical Systems*, Springer, 2001.
2. J. Guckenheimer, P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer-Verlag, New York, 1983.
3. S. Wiggins, *Introduction to Applied Nonlinear Dynamical Systems and Chaos*, Springer-Verlag, New York, 1990.
4. L. C. Evans, *Partial Differential Equations, Graduate Studies in Mathematics*, Vol. 19, American Mathematical Society, Providence, 1998.
5. R. C. McOwen, *Partial Differential Equations - Methods and Applications*, Pearson Education Inc., Indian Reprint 2004.
6. S. J. Farlow, *Partial Differential Equations for Scientists and Engineers*, Dover Publications, New York, 1982.

Course Title: Potential Flow of Fluids and Water-Wave Theory

Course Code: PHDMS.715

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Unit I

(16 Lecture Hours)

Real fluids and ideal fluids, Velocity of fluid at a point, Streamlines, Path lines, Streak lines, Velocity potential, Vorticity vector, Local and particle rate of change, Equation of continuity, Irrigational and rotational motion, Acceleration of fluid, Conditions at rigid boundary.

Unit II

(14 Lecture Hours)

Euler's equation of motion, Bernoulli's equation and their applications, Potential theorems, Axially symmetric flows, Impulsive motion, Kelvin's Theorem of circulation, Equation of vorticity.

Unit III

(16 Lecture Hours)

Equations of Motion. Two dimensional flow. Navier-Stokes equation of motion. Velocity potential and Laplace equation. Simple irrotational flows. Separation of variables for an axisymmetric flow. Bernoulli equation for unsteady irrotational flow. Deep water wave. Shallow water wave.

Unit IV

(14 Lecture Hours)

Theory of surface wave, Finite amplitude wave, One dimensional tidal dynamics, Linear and non-linear diffraction theory, Permutation methods. Water wave interaction with submerged spherical structures and floating cylinders, Solitary waves, Cnoidal wave, Schrodinger equation.

Recommended Books:

1. G. K. Batchelor, *An Introduction to Fluid Dynamics*, Cambridge Mathematical Library, 2005.
2. O. M. Phillips, *The Dynamics of Upper Ocean*, Cambridge University Press, 1977.
3. J. J. Stoker, *Water Waves*, Wiley- Blackwell, 1992.

Course Title: Topology
Course Code: PHDMS.716
Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Objective: The course is an introductory course on point-set topology. It is designed in such a way that the students can have deep knowledge in general topology and can understand further deeper topics in Differential/Algebraic Topologies and their allied areas.

Unit I **(16 Lecture Hours)**

Topological Spaces: Open sets, Closed sets, Neighborhoods, Bases, Sub bases, Limit points, Closures, Interiors, Continuous functions, Homeomorphisms. Examples of topological spaces: Subspace topology, Product topology, Metric topology, Topological manifolds. Quotient Topology: Construction of cylinder, Cone, Möbius band and Torus.

Unit II **(17 Lecture Hours)**

Connected spaces, Connected subspaces of the real line, Components and path components, Local connectedness. Compact spaces, Sequentially compact spaces, Heine-Borel Theorem, Compact subspaces of the real line, Limit point compactness, Local –compactness and one point compactification.

Unit III **(15 Lecture Hours)**

The Countability Axioms: Separable spaces, Lindelof spaces. Separation Axioms: Hausdorff spaces, Regularity, Complete regularity, Normality, Urysohn Lemma, Urysohn Metrization Theorem, Tietze Extension Theorem. Tychonoff Theorem.

Unit-IV **(12 Lecture Hours)**

The Stone-Čech Compactification, Complete metric spaces, Compactness in metric spaces, Pointwise and compact convergence, Ascoli’s Theorem and Baire spaces.

Recommended Books:

1. J. R. Munkres, *Topology- A First Course*, Prentice Hall of India, New Delhi, 1975.
2. M. A. Armstrong, *Basic Topology*, Springer, Paperback Edition, 2004.
3. K. D. Joshi, *Introduction to General Topology*, Wiley Eastern, Delhi, 1986.
4. M. G. Murdeshwar, *General Topology*, Wiley Eastern, New Delhi, 1983.
5. G. F. Simmons, *Introduction to Topology & Modern Analysis*, McGraw Hill, Auckland, 1963.
6. James Dugundji, *Topology*, Universal Book Stall, New Delhi, 1990.
7. S. Kumaresan, *Topology of Metric Spaces*, second edition, Narosa Publishing House New Delhi, 2015.
8. S. Willord, *General Topology*, Philippines: Addison Wesley Publishing Company, 1970.

Course Title: Differential Geometry of Curves and Surfaces

Course Code: PHDMS.717

L	T	P	Credits	Marks
4	0	0	4	100

Total Hours: 60

Objective: To introduce students to the local and global theory of curves and surfaces so that they can be enabled for further studies and research in Differentiable Manifolds, Differential Topology, Algebraic Topology, Riemannian Geometry.

Unit-I

(15 Lecture Hours)

Curves in Plane and Space: Parameterized curves, Tangent vector, Arc length, Reparametrization, Regular curves, Curvature and Torsion of smooth curves, Frenet-Serret formulae, arbitrary speed curves, Frenet approximation of a space curve. Osculating plane, Osculating circle, Osculating sphere, Involutives and Evolutives, Bertrand curves, Spherical indicatrices, Helices, Fundamental theorem of space curves,

Unit-II

(15 Lecture Hours)

Isometries of R^3 , Congruence of curves. Surfaces in R^3 : Definition and examples, Smooth surfaces, Tangent, Normal and Orientability. Examples of surfaces: Generalized cylinder and generalized cone, Ruled surfaces, Surface of revolution and Quadric surfaces. First fundamental form, Isometry of surfaces, Conformal mapping of surfaces, Surface Area, Equi-area maps and a Theorem of Archimedes,

Unit-III

(15 Lecture Hours)

Second fundamental form, Curvature of curves on a surface, Normal and Principal curvatures, Meusnier's theorem, Euler's theorem, Weingarten equations and Weingarten map, Geometric interpretation of principal curvatures, Umbilical points. Gaussian and Mean curvature, Pseudo Sphere, Flat surfaces, Surfaces of constant mean curvature, Gaussian curvature of compact surfaces, Gauss map.

Unit-IV

(15 Lecture Hours)

Geodesics: Definition and basic properties, Geodesic equations, Geodesics on a surfaces of revolution, Clairaut's theorem, Geodesics as shortest paths, Geodesic coordinates, Gauss Theorem, Egregium, Gauss equations, Codazzi-Mainardi equations, Compact surfaces of constant Gaussian curvature.

Recommended Books:

1. A. Pressley, *Elementary Differential Geometry*, Springer (Undergraduate Mathematics Series), 2001.
2. M. P. Do Carmo, *Differential Geometry of Curves and Surfaces*, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1976.
3. A. Gray, *Differential Geometry of Curves and Surfaces*, CRC Press, 1998.
4. B. O'Neill, *Elementary Differential Geometry*, Academic Press, 1997.
5. C. Bär, *Elementary Differential Geometry*, Cambridge University Press, 2001.
6. R. S. Millman & G. D. Prkar, *Elements of Differential Geometry*, Prentice Hall, Englewood, Cliffs, NJ, 1977.
7. T. J. Willmore, *An Introduction to Differential Geometry*, Oxford University Press, London, 1965.
8. D. Somasundaram, *Differential Geometry, A First Course*, Narosa Publishing House, New Delhi, 2005.

Course Title: Differential Topology

Course Code: PHDMS.718

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Objective: To introduce students to the basics of Differential Topology so that they are able to appreciate better the topics covered in allied courses like Algebraic Topology, Riemannian geometry and Riemann-Finsler geometry as well as be adequately prepared for pursuing research in these topics.

UNIT-I

(15 Lecture Hours)

Topological manifolds, Charts, Atlases, Smooth manifolds, Examples of smooth manifolds, Manifolds with boundary, Smooth functions on a manifold, Smooth maps between manifolds, Diffeomorphism, Smoothness in terms of components, Examples of smooth maps, Partial derivatives, and the Inverse function theorem.

UNIT-II

(15 Lecture Hours)

Tangent space and tangent bundle, The Differential of a map, Chain rule, Bases for the tangent space at a point, Curves in a manifold, Submersions, Immersions and embeddings, Smooth covering maps, Critical and regular points, Submanifolds, Rank of a smooth map, Submersion and immersion theorems, Bump functions and partition of unity, Sard's theorem, The Whitney embedding theorem(statement only).

UNIT-III

(15 Lecture Hours)

Vector fields and Lie bracket. Topological groups, Lie groups: Definition and examples, The product of two Lie groups, Lie subgroups, One parameter subgroups and exponential map, Homomorphism and isomorphism in Lie groups, Lie transformation groups, The tangent space and Left invariant vector fields of a Lie group.

UNIT-IV

(15 Lecture Hours)

Tensor algebra, Differential forms, Cotangent spaces, pullback of 1-forms, k-forms, Exterior product, Differential forms on a circle, Exterior derivative, Exterior algebra and Lie derivative, Global formulas for the Lie and exterior derivatives.

Recommended Books:

1. J. M. Lee, *Introduction to Smooth Manifolds*, GTM, Vol. 218, Springer, New York, 2003.
2. L. W. Tu, *An Introduction to Manifolds*, Second edition, Springer, 2011.
3. S. Kumaresan, *A Course in Differential Geometry and Lie Groups (Texts and Readings in Mathematics)*, Hindustan Book Agency, 2002.

Suggested Readings:

1. S. S. Chern, W. H. Chen and K. S. Lam, *Lectures on Differential Geometry*, World Scientific Publishing Co. Pvt. Ltd., 2000.
2. W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian Geometry*, 2nd edition, Academic Press, New York, 2003.
3. N. J. Hicks, *Notes of Differential Geometry*, D. Van Nostrand Reinhold Company, New York, 1965.
4. L. Conlon, *Differentiable Manifolds*, 2nd edition, Birkhauser Boston, Cambridge, MA, 2001.
5. F. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, Springer, New York, 1983.

Course Title: Algebraic Topology

Course Code: PHDMS.719

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Objective: The objective of this course is to introduce the student's concept in Algebraic topology so that they can pursue research in this field and its allied areas.

Unit-I (14 Lecture Hours)

The Fundamental group: Homotopy of paths, Homotopy classes, The Fundamental group, Change of base point, Topological invariance, Covering spaces, The Fundamental group of the circle.

Unit-II (14 Lecture Hours)

Retractions and fixed points, No Retraction Theorem, The Fundamental theorem of Algebra, The Borsuk-Ulam theorem, The Bisection theorem, Deformation Retracts and Homotopy type, Homotopy invariance.

Unit-III (16 Lecture Hours)

Direct sums of Abelian Groups, Free products of groups, Uniqueness of free products, Least normal subgroup, Free groups, Generators and relations, The Seifert-Van Kampen theorem, The Fundamental group of a wedge of circles.

Unit-IV (16 Lecture Hours)

Classification of covering spaces: Equivalence of covering spaces, The general lifting lemma, The universal covering space, Covering transformation, Existence of covering spaces.

Recommended Books:

1. James R. Munkres, *Elements of Algebraic Topology*, Perseus Books, 1995.
2. A. Hatcher, *Algebraic Topology*, Cambridge University Press, 2002.
3. Satya Deo, *Algebraic Topology: A Primer (Texts and Readings in Mathematics)*, Hindustan Book Agency, 2003.
4. M. A. Armstrong, *Basic Topology*, UTM Springer, 2000.
5. E. H. Spanier, *Algebraic Topology (2nd edition)*, Springer-Verlag, New York, 2000.
6. J. J. Rotman, *An Introduction to Algebraic Topology, Text in Mathematics, No. 119*, Springer, New York, 2004.
7. W. S. Massey, *A Basic Course in Algebraic Topology*, SPRINGER (SIE), 2007.
8. M. J. Greenberg and J. R. Harper, *Algebraic Topology: A First Course*, 2nd Edition, Addison-Wesley Publishing Co, 1997.

Course Title: Riemannian Geometry

Course Code: PHDMS.720

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Objective: The objective of the course is to introduce the basic concepts of Differentiable manifolds and Riemannian geometry and also for further studies and research in Riemannian geometry, Finsler geometry, Mathematical Physics and their applications in allied areas.

Unit I (16 Lecture Hours)

Differentiable manifolds, Examples: Spheres (in higher dimensions), Product of manifolds (construction of cylinder, Torus, n-torus as a product of manifolds). Smooth maps on a manifold, smooth map between two manifolds. Tangent space, Tangent bundle, Vector fields and Lie bracket. Covariant differentiation of vector fields and affine connection. Riemannian metric, Riemannian manifolds, Fundamental Theorem of Riemannian Geometry via Koszul Formula.

Unit II (14 Lecture Hours)

Tensors and tensor fields (Riemannian metric as the most significant example), Tensorial property, Covariant differentiation of tensor fields, Riemann curvature tensor, Ricci tensor, Definition of sectional, Ricci and scalar curvatures, Isometries, notion of covering spaces, pull-back metrics via diffeomorphisms.

Unit III (16 Lecture Hours)

Covariant differentiation of a vector field along a curve with specific examples, Arc length and energy of a piecewise smooth curve, Geodesics as length minimizing curves, First variation of arc length, To show that geodesics are critical points of the fixed end point first variation formula, Exponential map, Geodesic completeness, Geodesic normal coordinates, Hopf-Renow Theorem (statement only), Geodesic variations, Jacobi fields and Gauss lemma.

Unit IV (14 Lecture Hours)

Second variation formula, the index form (Jacobi fields as minimizers of the Index form), Global differential geometry, spaces of constant sectional curvature, Bonnet-Myers, Cartan-Hadamard Theorem, Cartan's Theorems (on determination of metric by curvature).

Recommended Books:

1. S. S. Chern, W. H. Chen and K. S. Lam, *Lectures on Differential Geometry*, World Scientific Publishing Co. Pte. Ltd., 2000.
2. W. M. Boothby, *An Introduction to Differentiable Manifolds and Riemannian geometry*, 2nd Edition, Academic Press, New York, 2003.
3. N. J. Hicks, *Notes of Differential Geometry*, D. Van Nostrand Reinhold Company, New York, 1965.
4. Marcel Berger, *A Panoramic View of Riemannian Geometry*, Springer; 1st Edition, 2003. Corr. 2nd printing 2007.
5. M. P. Docarmo, *Riemannian geometry*, Birkhauser Boston, 1992.
6. J. M. Lee, *Riemannian Manifolds: An Introduction to Curvature*, GTM, Springer, 1st Edition, 1997.
7. B. O' Neill, *Semi-Riemannian geometry with Applications to Relativity*, Academic Press, New York, 1983.

Course Title: Riemann Finsler Geometry

Course Code: PHDMS.721

L	T	P	Credits	Marks
4	0	0	4	100

Total Hours: 60

Objective: The objective of this course is to enable the students, concepts of Riemann-Finsler geometry so that they can pursue research in this area.

UNIT I

(15 Lecture Hours)

Minkowski norms, Euler's theorem, Fundamental inequality and its interpretation, Finsler structures: definitions and conventions, Examples: Minkowski and locally Minkowski spaces, Riemannian manifolds, Randers spaces, Berwald spaces, Finsler spaces of constant flag curvature. Fundamental metric tensor and Cartan tensor.

UNIT II

(15 Lecture Hours)

Vector bundle, Nonlinear connection on slit tangent bundle, Chern connection, structure equations, Horizontal and vertical covariant derivatives, hh -, hv -, vv -curvatures, Bianchi identities and their consequences, Ricci identities, Geodesic spray coefficients, Flag curvature: Definition, example and its predecessor, Schur's lemma.

UNIT III

(15 Lecture Hours)

Rund's differential equation and its consequence, Criterion for strong convexity, Berwald frame, Moore frame, Geometrical setup on sphere bundle, Cartan scalar(I), Landsberg scalar(J) and Gaussian curvature(K), Riemannian arc length of indicatrix, Gauss Bonnet theorem for Landsberg surfaces.

UNIT IV

(15 Lecture Hours)

Geodesics: Sprays, shortest paths. Projectively equivalent Finsler metrics, Projectively flat metrics, Parallel vector fields, Parallel translations, Berwald metrics, Landsberg metrics, Distortion and S-curvature, Randers metric of isotropic S-curvature.

Recommended Books:

1. D. Bao, S. S. Chern, Z. Shen, *An Introduction to Riemann Finsler Geometry*, Graduate texts in Mathematics 200, Springer-Verlag, New York, 2000.
2. S. S. Chern and Z. Shen, *Riemann Finsler Geometry*, Nankai Tracts in Mathematics, Vol. 6. World Scientific Publishing Co. Pvt. Ltd., 2005.
3. Z. Shen, *Lectures on Finsler geometry*, World Scientific Press, 2001.

Further Readings:

1. M. Matsumoto, *Foundations of Finsler Geometry and Special Finsler Spaces*, Kaisheisha press, Saikawa, Otsu, 520, Japan, (1986).
2. P. L. Antonelli (ed.), *Handbook of Finsler Geometry*, Kluwer Academic Publishers, Dordrecht, The Netherlands, 2003.
3. Xinyue Cheng and Zhongmin Shen, *Finsler geometry-An Approach via Randers spaces*, First Edition, Springer Berlin Heidelberg, 2012.

Course Title: Lie Groups and Lie Algebra

Course Code: PHDMS.722

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Objective:

The aim of this course is to make the students learn basic concepts of Lie groups and Lie algebra, so as to enable the students to understand further topics related to solution of differential equations.

Unit I **(15 Lecture Hours)**

Differential Manifolds, Topological manifolds, Charts, Atlases and smooth structure, Smooth maps and diffeomorphism, Partitions of Unity, Tangent space, Tangent map, Vector fields and 1-forms.

Unit II **(15 Lecture Hours)**

Lie Groups Definition and examples, Linear Lie groups, Lie group homomorphism, Lie algebra and the exponential map, Adjoint representation, Homogeneous spaces, Baker-Campbell-Housdorff formula.

Unit III **(15 Lecture Hours)**

Lie Algebras Definition and examples, Classical Lie algebras, Solvable and nilpotent Lie algebras, Lie and Engel theorems, Semi-simple and reductive algebras, Semi-simplicity of Classical Lie algebras

Unit IV **(15 Lecture Hours)**

Semisimple Lie algebras; Killing form; Jordan decomposition; Engel's Theorem, Cartan subalgebra and Root space decomposition, Geometry of Root systems, Simple roots and Weyl group, Classification of root systems; Examples

Recommended Books:

1. J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Graduate Text in Mathematics, 9, Springer-Verlag, 1980.
2. N. Jacobson, *Lie Algebras*, Wiley-Interscience, New York, 1962.
3. J. P. Serre, *Lie Algebras and Lie Groups*, Benjamin, New York, 1965.
4. N. Bourbaki, *Lie Groups and Lie Algebras*, Springer Science & Business Media, 1998.
5. K. J. Alexander, *An Introduction to Lie Groups and Lie Algebras*, Cambridge University Press.
6. S. Kumaresan, *Differential Geometry and Lie Groups*, Hindustan Book Agency.
7. B. Hall, *Lie Groups, Lie Algebras, and Representations: An Elementary Introduction*, Second Edition, Springer.
8. P. J. Olver, *Application of Lie Groups to Differential Equations*, Second Edition, Springer.

Course Title: Probability Theory

Course Code: PHDMS.723

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Objectives:

The course is designed to equip the students with various probability distributions and to develop greater skills and understanding of Sampling and Estimation.

Unit I

(13 Lecture Hours)

Probability: Definition of probability-classical, Relative frequency, Statistical and axiomatic approach, Addition theorem, Boole's inequality, Conditional probability and multiplication theorem, Independent events, Mutual and pairwise independence of events, Bayes' theorem and its applications.

Unit II

(14 Lecture Hours)

Random Variable and Probability Functions: Definition and properties of random variables, Discrete and continuous random variables, Probability mass and density functions, Distribution function. **Concepts of Bivariate Random Variable:** Joint, Marginal and conditional distributions.

Mathematical Expectation: Definition and its properties. Variance, Covariance, Moment generating function- Definitions and their properties. Chebychev's inequality.

Unit III

(14 Lecture Hours)

Discrete distributions: Uniform, Bernoulli, Binomial, Poisson and geometric distributions with their properties.

Continuous distributions: Uniform, Exponential, Gamma and normal distributions with their properties. Central Limit Theorem (Statement only).

Statistical estimation: Parameter and statistic, Sampling distribution and standard error of estimate. Point and interval estimation, Unbiasedness, Efficiency.

Unit IV

(15 Lecture Hours)

Sampling Theory: Types of sampling, Errors in sampling, Parameter and statistic, **Tests of Significance:** Null Hypothesis, Alternative Hypothesis, One-tailed, Two-tailed tests. Sampling Attributes: Tests of Significance for single proportion and difference of proportions. Sampling of Variables.

Recommended books:

1. A. M. Mood, F. A. Graybill and D. C. Boes, *Introduction to the Theory of Statistics*, Mc Graw Hill Book Company, 1966.
2. J. E. Freund., *Mathematical Statistics*, Prentice Hall of India.
3. S. C. Gupta and V. K. Kapoor, *Fundamentals of Mathematical Statistics*, S. Chand Pub., New Delhi.
4. M. Spiegel, *Probability and Statistics*, Schaum Outline Series.
5. S. M. Ross., *Introductory Statistics*, 2003.
6. R. V. Hogg and T. T. Craig, *Introduction to Mathematical Statistics* (4th Edition), Collier-McMillan, 1978.

Course Title: Stochastic Processes and Queuing Theory

Course Code: PHDMS.724

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Objective: The course on Stochastic Processes and Queuing Theory, is framed to equip the students with knowledge of terms involved in Stochastic Processes as well as concepts and measures in Queuing Theory.

Unit I **(15 Lecture Hours)**

Review of probability, Random variables and distributions, Generating functions and transforms; Stochastic processes, Discrete and continuous-time Markov chains, Renewal processes,

Unit II **(15 Lecture Hours)**

Brownian motion; Characteristics of queueing systems, Little's formula, Markovian and non-Markovian queueing systems, Embedded Markov chain applications to M/G/1, G/M/1, and related queueing systems,

Unit III **(15 Lecture Hours)**

Queues with vacations, Priority queues, Queues with modulated arrival process, Discrete-time queues and matrix-geometric methods in queues; Networks of queues, Open and closed queueing networks,

Unit IV **(15 Lecture Hours)**

Algorithms to compute the performance metrics; Simulation of queues and queueing networks; Application to manufacturing, Computer and communication systems and networks.

Recommended Books:

1. L. Kleinrock, *Queueing Systems*, Vol. 1: Theory, 1975, Vol. 2: Computer Applications, 1976, John Wiley and Sons.
2. J. Medhi, *Stochastic Models in Queueing Theory* (2nd Edition), Academic Press, 2002.
S. Asmussen, *Applied Probability and Queues* (2nd Edition), Springer, 2003.
3. D. Gross, and C.Harris, *Fundamentals of Queueing Theory*, 3rd Edition, John Wiley and Sons, 1998.
4. R.B. Cooper, *Introduction to Queueing Theory* (2nd Edition), North-Holland, 1981.
5. R. Nelson, Probability, *Stochastic Processes, and Queueing Theory: The Mathematics of Computer Performance Modelling*, Springer-Verlag, 1995.
6. E. Gelenbe, and G. Pujolle, *Introduction to Queueing Networks* (2nd Edition), John Wiley, 1998.

Course Title: Reliability Theory

Course Code: PHDMS.725

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Objectives:

The course on Reliability Theory is framed to equip the students with knowledge of terms involved in reliability theory as well as concepts and measures.

Unit I

(15 Lecture Hours)

Reliability Concepts and Measures: Components and systems, Coherent systems, Reliability of coherent systems, Cuts and paths, Modular decomposition, Bounds on system reliability, Structural and reliability importance of components.

Unit II

(15 Lecture Hours)

Life distributions and associated survival, Conditional survival and hazard rate functions. Exponential, Weibull, Gamma life distributions and estimation of their parameters.

Unit III

(15 Lecture Hours)

Notions of ageing. IFR, IFRA, NBU, DMRL, NBUE, and HNBUE classes; their duals and relationships between them. Closures of these classes under formation of coherent systems, convolutions and mixtures.

Unit IV

(15 Lecture Hours)

Partial orderings: Convex, star, stochastic, failure rate and mean-residual life orderings. Univariate shock models and life distributions arising out of them. Maintenance and replacement policies, Availability of repairable systems.

Recommended Books:

1. R. E. Barlow and F. Proschan, *Statistical Theory of Reliability and Life Testing*, Holt, Rinehart and Winston, 1985.
2. J. F. Lawless, *Statistical Models and Methods of Life Time Data*, John Wiley Models, Marcel Dekker, 1982.
3. M. Shaked and J. G. Shanthikumar, *Stochastic Orders & Their Applications*, Academic Press, 1994.

Course Title: Sampling Theory

Course Code: PHDMS.726

Total Hours: 60

L	T	P	Credits	Marks
4	0	0	4	100

Objectives:

The course is designed to equip the students with basic knowledge of different sampling schemes, their mean and variance estimations and also give understanding of non-sampling errors.

Unit I

(15 Lecture Hours)

Introduction to usual notations used in sampling. Basic finite population sampling techniques: SRSWOR, SRSWR, stratified, systematic and related results on estimation of population mean/total. Relative precision of different sampling techniques. Allocation problem in stratified sampling.

Unit II

(15 Lecture Hours)

Ratio and regression estimators based on SRSWOR method of sampling. Two-stage sampling with equal size of first stage units. Double sampling for ratio and regression methods of estimation. Cluster sampling - equal clusters.

Unit III

(15 Lecture Hours)

PPS WR/WOR methods [cumulative total, Lahiri's schemes] and related estimators of a finite population mean [Thompson-Horwitz, Yates and Grundy estimator, Desraj estimators for a general sample size and Murthy's estimator for a sample of size 2].

Unit IV

(15 Lecture Hours)

Sampling and Non-sampling error with special reference to non-response problems. National sample surveys organization (NSSO) and role of various statistical organizations in national development.

Recommended Books:

1. A. Chaudhuri, *Essentials of Survey Sampling*, Prentice Hall of India, 2010.
2. A. Chaudhari and H. Stenger, *Survey Sampling Theory and Methods*, 2nd Edition, Chapman and Hall, 2005.
3. W. G. Cochran, *Sampling Techniques*, John Wiley & Sons, New York, 1977.
4. A. S. Hedayat and B. K. Sinha, *Design and Inference in Finite Population Sampling*, Wiley, New York, 1991.
5. P. S. Levy and S. Lemeshow, *Sampling of Populations-Methods and Applications*, Wiley, 2008.
6. S. K. Thompson, *Sampling*, John Wiley and Sons, New York, 2002.
7. P. Mukhopadhyay, *Theory and Methods of Survey Sampling*, Prentice Hall of India, 1998.